King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 302  Major Exam II
The Third Semester of 2018-2019 (183)

Time Allowed: 120 Minutes

Name: ___________________________  ID#: __________________
Section/Instructor: _________________  Serial #: _______________

- Mobiles and calculators are not allowed in this exam.
- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. No points for answers without justification.

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Q:1 (15 points) Convert $\hat{A} = y\hat{a}_x + (y + z)\hat{a}_y$ into spherical coordinates. Evaluate $\hat{A}$ at the point $(1, \frac{\pi}{2}, \frac{\pi}{4})$. 
Q:2 (15 + 5 = 20 points) (a) Find the directional derivative of \( V = \ln \rho^2 \sin^2 \phi \ z^3 \) in the direction of \( \hat{A} = \hat{a}_x + \hat{a}_y + \hat{a}_z \) at the point \( P(1, \frac{\pi}{4}, 2) \).

Hint: Convert \( \hat{A} \) into Cylindrical coordinates.

(b) Find a unit vector tangent to \( V \) at \( P \)
Let $\vec{E} = x^2 y \hat{a}_x - y \hat{a}_y$.

(a) Find $\oint_L \vec{E} \cdot d\vec{l}$, where $L$ is the closed curve: $y = x^2$ from $(0,0)$ to $(1,1)$ and $y = x$ from $(1,1)$ to $(0,0)$.

(b) Find $\iint_S (\nabla \times \vec{E}) \cdot d\vec{S}$, where $S$ is the area bounded by $L$ and $y = x$.

(c) What theorem is verified from (a) and (b).
Cont.......
Q:4 (15 points) Use Stoke’s theorem to find flux of the curl of the vector field

\[ \vec{A} = \frac{1}{r^2} \cos \theta \, \hat{a}_r + r \sin \theta \cos \phi \, \hat{a}_\theta + \cos \theta \, \hat{a}_\phi \]

through the hemisphere \( r = 1 \) and \( 0 \leq \theta \leq \frac{\pi}{2} \).
Q:5 (15 points) Let $\hat{A} = \rho^2 \cos \phi \hat{a}_\rho + z \sin \phi \hat{a}_\phi + \rho z \hat{a}_z$.

Use the divergence theorem to calculate the flux of $\hat{A}$ over the closed surface of the cylinder $0 \leq z \leq 4, \rho = 2$. 
Q:6 (15 points) A point charge of 10 \( nC \) is located at \((5, 0, 0)\), while line \( y = 2, z = 1 \) carries a uniform charge of 15 \( nC \). If the potential at \( A(1, 4, 3) \) is 15V, find the potential at \((0, 0, 0)\).