1. Eric deposits 80 into a savings account at time 0, which pays interest at an annual nominal rate of \( i \), compounded semiannually. Mike deposits 150 into a different savings account at time 0, which pays simple interest at an annual rate of \( i \). Eric and Mike earn the same amount of interest during the last 6 months of the 7th year. Calculate \( i \).

2. A 10-year loan of 1000 is repaid with payments at the end of each year. Each of the first five payments equals 200% of the amount of interest due. Each of the last five payments is \( X \). The lender charges interest at an annual effective rate of 10%. Calculate \( X \).

3. A loan is amortized over five years with monthly payments at an annual nominal interest rate of 12% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 4% lower than the prior payment. Calculate the outstanding loan balance immediately after the 30th payment is made.

4. Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of \( \frac{i}{2} \). The price assumes an annual nominal yield of \( \frac{i}{2} \), compounded semi-annually. As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of \( i \). At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond. Calculate \( i \).

5. A common stock pays a constant dividend at the end of each year into perpetuity. Using an annual effective interest rate of 5%, calculate the Macaulay duration of the stock.

6. Joe must pay liabilities of 2000 due one year from now and another 1000 due two years from now. He exactly matches his liabilities with the following two investments:
   Mortgage I: A one year mortgage in which \( X \) is lent. It is repaid with a single payment at time one. The annual effective interest rate is 5%.
   Mortgage II: A two-year mortgage in which \( Y \) is lent. It is repaid with two equal annual payments. The annual effective interest rate is 6%.
   Calculate \( X + Y \).

7. A liability consists of a series of 10 annual payments of 20,000 with the first payment to be made one year from now.
   The assets available to immunize this liability are three-year and five-year zero-coupon bonds.
   The annual effective interest rate used to value the assets and the liability is 5%. The liability has the same present value and duration as the asset portfolio. Calculate the amount invested in the five-year zero-coupon bonds.

8. On January 1, an investment fund was opened with an initial balance of 4000. Just after the balance grew to 4200 on July 1, an additional 2000 was deposited. The annual effective yield rate for this fund was 10% over the calendar year. Calculate the time-weighted rate of return for the year.

9. Let \( S \) be the accumulated value of 500 invested for two years at a nominal annual rate of discount \( d \) convertible semiannually, which is equivalent to an annual effective interest rate of \( i \). Let \( T \) be the accumulated value of 500 invested for one year at a nominal annual rate of discount \( d \) convertible quarterly. \( S/T = (29/28)^4 \). Calculate \( i \).
10. A borrower takes out a loan of 4000 at an annual effective interest rate of 6%.
Starting at the end of the fifth year, the loan is repaid by annual payments, each of which
equals 600 except for a final balloon payment that is less than 1000.
Calculate the final balloon payment.

11. On January 1, a fund is worth 80,000. On June 1, the value has increased to 100,000
and then 30,000 of new principal is deposited. On October 1, the value has declined to
120,000 and then 40,000 is withdrawn. On January 1 of the following year, the fund is
again worth 80,000. Calculate the dollar-weighted rate of return using the simple interest
approximation.

12. A common stock will pay 3 per share in dividends at the end of the current year. You are
given that the earnings of the corporation increase 6% per year indefinitely, the number
of shares increases 3% per year indefinitely, and that the corporation plans to continue to
pay the same percentage of its earnings in dividends. The price of the stock 10 years from
the beginning of the current year will be $X$. At that time, the annual effective interest
rate is assumed to be 5%. Calculate $X$ using the dividend discount model.

13. A 20-year bond priced to have an annual effective yield of 8% has a Macaulay duration
of 10. Immediately after the bond is priced, the market yield rate increases by 0.25%.
Calculate the bond’s approximate percentage price change, using a first-order modified
approximation and first-order Macaulay approximation.

14. Anderson Bank offers a six-year loan. It is repaid with a single payment of principal
and interest at time six. Anderson wants to receive an annual rate of 10% compounded
continuously to reflect deferred compensation. For this loan, the percentage of borrowers
that will default is 1%. For the loans where there are defaults, Anderson Bank will be able
to recover 40% of the amount owed after six years. Let $\delta$ be the credit spread calculated as
an annual rate compounded continuously that Anderson Bank needs to charge. Calculate
$\delta$.

15. You are given the following spot interest rates:

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5%</td>
</tr>
<tr>
<td>2</td>
<td>3.8%</td>
</tr>
<tr>
<td>3</td>
<td>4%</td>
</tr>
<tr>
<td>4</td>
<td>4.2%</td>
</tr>
<tr>
<td>5</td>
<td>4.4%</td>
</tr>
</tbody>
</table>

John purchases a deferred interest rate swap with a term of five years. Under the swap,
there is no swapping of interest rates during the first two years. During the last three
years, the settlement period will be one year. Under this swap, John will be the payer.
The variable interest rate will be based on the one year spot rate at the start of each
settlement period. The notional amount of this swap is 500,000. Calculate the swap rate
for this swap.
Eric's interest in the last 6 months of the 7th year is 
\[ 80 \left(1 + \frac{1}{2}\right)^{\frac{13}{2}} \]. Mike's simple interest for the same period is 150 \( \frac{1}{2} \).

\[ 80 \left(1 + \frac{1}{2}\right)^{\frac{13}{2}} = 150 \frac{1}{2} \]

\[ \left(1 + \frac{1}{2}\right)^{\frac{13}{2}} = \frac{15}{8} \quad i = 9.9\% \]

For the first 5 years, each payment is 200% of interest due. The lender charges 10% therefore 10% of the principal outstanding will be used to reduce the principal.

At the end of 5 years the amount outstanding is 
\[ 1000 \left(0.9\right)^5 = 590.49 \].

Thus, the equation of value for the last 5 years is 
\[ 590.49 = X a_{\overline{5}|0.1} = 3.790X \quad X = 155.80 \]

Monthly payment at time \( t \) is 1000 \( 0.96 \)^{t-1}.
We will calculate the balance prospectively.
The value at time 30 months is the PV of payments from time 31 to time 60.

\[ OB_{30} = 1000 \left(0.96^{30} \_ + \ldots + 0.96^{59} \_ 39\right) \]

\[ = 1000 \left(\frac{0.96^{30} - 0.96^{10} \_ 31}{1 - 0.96 \_} \right), \quad \nu = \frac{1}{1.01} \]

\[ = 4595.82 \]

Because yield rate is equal to the coupon rate, Bill paid 1000 for the bond. In return he receives 20 every six months which accumulates to 40\( \frac{3}{2} \) every year, \( j \) is the semiannual interest rate. The equation of value is 
\[ 1000 \left(1.07\right)^{10} = 40 \left(\frac{3}{2} \right) + 1000 \]

\[ j = 0.084969 \]

\[ i = (1 + 0.084943)^{2} - 1 = 0.17714\% \]

The size of dividend doesn't matter, so assume it is 1.

Then the duration is 
\[ \frac{\sum_{t=1}^{\infty} t \nu^t}{\nu} = \frac{1}{d} = \frac{1}{\frac{1}{d}} = 1.25 = 21 \]

Because only Mortgage II provides a cash flow at time two, it must be considered first. The mortgage provides 
\[ Y a_{\overline{21}|0.06} = 0.54544Y = 1000 \]

\[ Y = 1833.40 \]

Mortgage I must provide 2000-1000 = 1000 at time one and thus 
\[ X = \frac{1000}{0.05} = 952.38 \]

\[ X + Y = 2,785.78 \]
7. The PV of the liability is \(20,000 \times 1.005^{10} = 154,434.70\).

The duration of the liability is

\[
\frac{\sum v^t \cdot R_t}{\sum v^t} = \frac{20,000 + 2(20,000) v^2 + \ldots + 10(20,000) v^{10}}{154,434.70} = 5.0991
\]

\(X\) is the amount invested.

\[
\frac{X}{154,434.70} \cdot 3 + \left(1 - \frac{X}{154,434.70}\right) \cdot 0.07 = 74,844.69
\]

\[X = 4,329,361.48\]

8. The ending balance is

\[4,000 \times (1.1) + 2,000 \times (1.1)^2 = 64,976.2\]

Time weighted return is

\[\frac{4,200 \times 64,976.2}{4,000 + 2,000} - 1 = 0.1004\]

9. \[\frac{(1 - \frac{d_2}{2})^{-4}}{(1 - \frac{d_4}{4})^{-4}} = (\frac{29}{28})^4 \quad 1 - \frac{d_2}{2} = \frac{28}{29} \]

\[29 - \frac{29}{2} \cdot d = 28 - \frac{11}{2} \cdot d \quad 1 - \frac{15}{2} \cdot d \]

\[d = \frac{2}{15}\]

\[1 + i = (1 - \frac{d_2}{2}) = (1 - \frac{11}{15})^{-2} \quad i = 0.147\]

10. \[4000 = 600 \times a_{11.06} = 8.4165 \quad n = 12.07\]

There are 11 regular payments. The equation for the balloon payment \(X\) is

\[400 = 600 v^x a_{11.06} + X v^x \Rightarrow X = 639.43\]

11. \[80,000(1+i) + 30,000(1 + \frac{i}{12}) - 40,000(1 + \frac{i}{4}) = 80000\]

\[10,000 = 87,500; \quad i = 0.1142\]

12. The present value of the dividend is:

\[
\frac{3 \times 1.06^{10}}{1.03^{10}} \left(\frac{1}{1.05} + \frac{1.06}{1.03 \cdot 1.05^2} + \ldots\right)
\]

\[= \frac{3 \times 1.06^{10}}{1.03^{10}} \times \frac{1}{1.05} \times \frac{1}{1 - 1.06/1.03 \cdot 1.05} = 191.51\]

13. Modified duration is \[\frac{10}{1.08} = 9.2593\]

\[P(0.0825) \approx P(0.08); (1 - (0.0825 - 0.08)) = 9.2593\]

\[\approx 0.97698(0.08)\]

\[\frac{100(0.97698 - 1)}{100(0.97698 - 1)} = -2.34\% \quad \text{[modified]}\]

\[P(0.0825) \approx P(0.08)(\frac{0.08}{0.0825})^{10} = 0.9771 P(0.08)\]

\[\frac{100(0.9771 - 1)}{100(0.9771 - 1)} = -2.285\% \quad \text{[Macaulay]}\]
Assume 1 is borrowed. Anderson wants to receive $e^{6 \times 0.1} = 1.822$.

The actual rate charged will be $0.1 + \delta$. The expected amount received, given the probability of default is

\[(1 - 0.01) e^{6(0.1+\delta)} + 0.04 \times 0.4 \times e^{6(0.1+\delta)} = 1.82212\]

\[0.994 e^{0.6 \times 6\delta} = 1.8112 e^{6\delta} = 1.82212\]

\[\delta = 0.0010\]

\[R = \frac{P_2 - P_5}{P_2 + P_4 + P_5} = \frac{(1.038)^{-2} - (1.044)^{-5}}{(1.04)^{-3} + (1.0492)^{-4} + (1.044)^{-5}}\]

\[= 0.04789\]