Question 1. A perpetuity paying 1 at the beginning of each 6-month period has a present value 20. A second perpetuity pays $X$ at the beginning of every 2 years. Assuming the same annual effective interest rate, the two present values are equal. Determine $X$.

\[ PV = \frac{1}{j_1} \]

\[ 20 = 1 \]

\[ j_1 = \frac{1}{20} = 0.05 \]

\[ j_1 \] is the interest rate for the first one.

\[ PV = \frac{X}{j_2} \]

\[ 20 = \frac{X}{0.2155} \]

\[ X = 4.31 \]

\[ 1+i = (1.05)^2 \]

\[ i = 0.1025 \]

\[ j_2 \] is the 2 years interest rate.

\[ (1+i) = (1+j_2)^{\frac{1}{2}} \]

\[ j_2 = 1.025 - 1 = 0.2155 \]

QUESTIONS 2 IS ON THE BACK OF THE PAGE.
Question 2. A perpetuity costs 77.1 and makes annual payments at the end of the year. The perpetuity pays 1 at the end of the year 2, 2 at the end of the year 3, ..., \( n \) at the end of the year \((n+1)\). After year \((n+1)\), the payment remain constant at \(n\). The annual effective interest rate is 10.5\%. Calculate \(n\).

\[
\begin{align*}
\text{PV} &= \frac{n}{i} - (D_1 a_{\frac{n}{i}}) + \frac{n}{i} \text{ or } (Ia_{\frac{n}{i}}) + \frac{n}{i} y^{n+1} \\
77.1 &= \frac{n}{0.105} - \frac{n}{0.105} a_{\frac{n}{0.105}} \\
0.0955 &= a_{\frac{n}{0.105}} = \frac{1-y^n}{0.105} \\
y^n &= 0.1499725 \\
(1.105)^{-n} &= 0.1499725 \\
n &= 19
\end{align*}
\]