Name_____________________________  ID#:_________________ Serial #:___

Instructions.
1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.
9. IMPORTANT. Keep answers to questions on probabilities and rates in at least 5 significant decimal points.

The test is 150 minutes, GOOD LUCK, and you may begin now!

<table>
<thead>
<tr>
<th>Question</th>
<th>Total Marks</th>
<th>Marks Obtained</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6+3+3=12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4+6=10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5+5+4 = 14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4+4=8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5+4=9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4+3 = 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4+4+5=13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Extra blank page
1. (10 marks) A discrete 500 000 benefit contingent contract is issued to the last-survivor status (\(\bar{y}\bar{y}\)) where the following hold:
   I. the two future lifetime random variables \(T_y\) are independent.
   II. the contract is funded by discrete net annual premiums, which are reduced by 25% after the first failure.
   III. \(A_y = 0.40\), \(A_{\bar{y}} = 0.55\), and \(\bar{a}_y = 10\).

   Under the equivalence principle, find the value of the initial net annual premium.
2. (6+3+3=12 marks) Students can leave a certain three-year school only for reasons of failure (decrement 1) or voluntary withdrawal (decrement 2), where each decrement is uniformly distributed over \((x, x+1)\) in its associated single-decrement table. The following values are given:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(q_x^{(1)})</th>
<th>(q_x^{(2)})</th>
<th>(q_x^{(1)})</th>
<th>(q_x^{(2)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculate

a) The **missing probability values** in the table above
b) The **marginal probability** \(p_k(1)\)
c) The probability that a person **who decremented** from school in the **third year** is decrementing due to **failure**.
3. (4+6=10 marks) The annual coupon yields in the table below are for annual-payment coupon bonds.

<table>
<thead>
<tr>
<th>Maturity (in Years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual yield for coupon-bearing bonds</td>
<td>2%</td>
<td>4%</td>
<td>6%</td>
<td>8%</td>
</tr>
<tr>
<td>Annual yield for zero-coupon bonds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Calculate the corresponding zero-coupon yields of the same maturities
b) Calculate forward rates that can be obtained from combination of the bonds above.
4. (5+5+4 = 14 marks) The following table provide $n$-year forward one-year rates.

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{n,1}$</td>
<td>4%</td>
<td>5%</td>
<td>6%</td>
<td>7%</td>
<td>8%</td>
</tr>
</tbody>
</table>

(Note: Keep at least 6 decimal places {or 4 decimals for percentages} for all calculations below)

a) Using the information in the table above and, find all determinable spot rates.

b) Complete the missing forward rates in the table below

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{n,n-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) A 5-year pure endowment contract issued to a person age 40 is funded with level annual premium and has a maturity benefit of SAR10 000. Premiums are paid at the beginning of each year. Using the mortality rates below and the relevant information given above, calculate the annual premium.

<table>
<thead>
<tr>
<th>$x$</th>
<th>40</th>
<th>41</th>
<th>42</th>
<th>43</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_x$</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
</tr>
</tbody>
</table>
5. (4+4 = 8 marks) For an equity-indexed UL contract, you are given:
   i. The face amount is 100 000
   ii. Annual point-to-point indexing method is used with
       a. 10% index cap
       b. 1% index floor
       c. 100% participation rate
   iii. Policy charges and expenses are deducted at the beginning of each year
       a. 4% premium expense rate
       b. An administrative charge of 50
   iv. The gross annual premium is $G = 1000$ is paid at start of each year
   v. Interest is credited at the end of each year
   vi. The following values apply over the next three years

<table>
<thead>
<tr>
<th>Year</th>
<th>Index Closing value</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>1080</td>
<td>1200</td>
<td>1100</td>
</tr>
<tr>
<td>Cost of Insurance per 1000 of Amount at risk</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surrender Charge per 1000 of Face Amount</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Find the **credited interest rate** for the contract at the end of each of the next three years
b) Find the **Cash value** of the contract at the end of each of the next three years.
6. (7 marks) Consider a two-year discrete term policy issued to (35) with the following details:
   i. A gross annual premium of 100
   ii. Profit margin of -10%
   iii. Profit vector, \( Pr = (-300, 75, 250) \)
   iv. \( q_{35} = 0.00278 \)

Find the value of the **risk discount** rate.

Solution (cf. MQR6 chap 17 Q17.3 pg474):

\[
\frac{1}{1 + r} \quad \Pi = (-300, 75, 250) \cdot (1 - 0.00278) = (-300, 75, 249.305)
\]

\[
NPV = \Pi_0 + \Pi_1 + r + \Pi_2 (1 + r)^2
\]

\[
\begin{align*}
APV & = G \left( 1 + \frac{1 - 0.00278}{1 + r} \right) \\
Profit Margin & = \frac{NPV}{APV} G
\end{align*}
\]

\[
\begin{align*}
-300 + 75 v + 249.305 v^2 & = -300 + 75 v + 249.305 v^2 \\
10 & = 1 - 9.9722 v & \\
& \rightarrow -10 - 9.9722 v & = -300 + 75 v + 249.305 v^2
\end{align*}
\]

\[
249.305 v^2 + 10 v - 290 = 0
\]

\[
v = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 249.305 \cdot (-290)}}{2 \cdot 249.305}
\]

\[
v = \frac{-10 \pm \sqrt{296414.07477284}}{498.61}
\]

\[
v = \frac{-10 \pm 1544.4392296416929}{249.305}
\]

\[
v = \frac{-20.9214958176564708}{249.305}
\]

\[
So, r = \frac{v - 1}{v} = \frac{-20.9214958176564708}{-19.9214958176564708} = 0.08519.
\]
7. (5+4=9 marks) For a discrete 5-year term life insurance on (45), you are given:
   i. The mortality assumption is $q_{45+t} = 0.013 + 0.001t$ for $t = 0, 1, 2, 3, 4$.
   ii. The interest assumption is $i = 0.05$.
   iii. Ignoring reserves, the profit vector is $Pr = (-300, 450, 230, 100, -50, -220)$

   The insurer wishes to set reserves by zeroization.
   
a) Calculate the sequence of zeroized reserves
   b) Calculate the new profit vector using the zeroized reserves.
8. (4+3 = 7 marks) Suppose that $s_x = 1.045^x$ for $x \geq 20$.
   a) A life aged exactly 36 at the valuation date receives 80 000 in salary in the year to the valuation date. Calculate his predicted average salary in his last 3 years of working if he plans to retire at age 65.
   b) A life aged exactly 35 has a current annual salary of 80 000. Estimate his salary between ages 55 and 56. Assume the life remains in employment until at least age 56 and salaries are revised each year, the last adjustment being 6 months before the valuation date.

Solution:

(a) The year to valuation date” means the year just before the life reaches age 36. The salary received in age 35 to 36 is 80 000.

The salary received in age 62 to 63 is $80000 \cdot s_{62} / s_{35}$.

The salary received in age 63 to 64 is $80000 \cdot s_{63} / s_{35}$.

The salary received in age 64 to 65 is $80000 \cdot s_{64} / s_{35}$.

The final average salary received in last 3 years is $80000 \cdot \left(1 + \frac{1}{1.045^{32}} + \frac{1}{1.045^{27}} \right) = 80000 \cdot 1.045^{27}$.2279.

(b) The salary received in year of age 34.5 to 35.5 is exactly 80 000. The salary between ages 55 and 56 is estimated to be $80000 \cdot s_{55} / s_{34.5} = 80000 \cdot 1.045^{20.5} = 197230.44$.2279.
9. (4+4+5=13 marks) An employer establishes a Defined Contribution (DC) plan. The contribution rate is set using the following assumptions:
   I. Contributions are deposited into the member’s account monthly in arrears at a fixed percentage of 10% of the salary rate at that time.
   II. Contributions earn investment returns of 10% effective per year.
   III. The salary scale is given by \( s_x = 1.05^x \) for \( 25 \leq x \leq 60 \) and salaries are assumed to increase continuously.

Karl is an employee aged 25. His contributions start this month. The current salary is 65 000 per year. Karl plans to retire at age 60. Upon retirement, he would use the proceeds to buy a life annuity immediate payable monthly.

a) Calculate the value of the projected fund at retirement.

b) Suppose that under an appropriate mortality and interest rate assumption, \( \ddot{a}^{(12)}_{60} = 14.44 \).
   Calculate the replacement ratio for Karl.

c) It turns out that the realized investment returns is only 9% effective per year. If the same post-retirement mortality and interest rate for the calculation of life annuity applies, what is the pension benefit per year for Karl?

Test Ends