King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 9 Quiz II (Term 191)

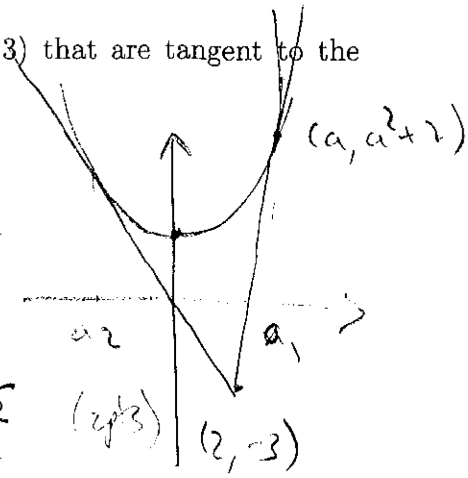
Name : ....KEY.......................... ID #.................................... Serial #: ....

(6 pts) 1. If $f(x) = x^{\pi} + 2\pi^{e} + 3x^{4} - 5 + 2e^{x}$ then find $f'(1)$.

$$f'(x) = \pi x^{\pi - 1} + 12x^{3} + 2e^{x}$$

$$\Rightarrow f'(1) = \pi + 12 + 2e$$

(4 pts) 2. Find equations of both lines through the points $(2, -3)$ that are tangent to the parabola $y = x^2 + 2$.



We will find the slope of the tangent line at a in two different ways. That means, we have

(1 pt)

$$y'\Big|_{x=a} = \frac{f(a) - (-3)}{a - 2} \Rightarrow 2a = \frac{a^2 + 5}{a - 2}$$

(2 pts)

$$\Rightarrow 2a^2 - 4a = a^2 + 5 \Rightarrow a^2 - 4a - 5 = 0 \Rightarrow (a-5)(a+1) = 0$$

$$\Rightarrow a = -1 \ \& \ a = 5.$$

(2 pts)

For $a = -1 \Rightarrow y = 3$ and $m = -2 \Rightarrow$ Equation

$$y - 3 = -2(x+1) \Rightarrow \boxed{y = -2x + 1}$$

For $a = 5 \Rightarrow y = 27$ and $m = 10 \Rightarrow$ Equation

$$y - 27 = 10(x - 5) \Rightarrow \boxed{y = 10x - 23}$$

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(pts) 3. Where does the normal line to the parabola $y = x - x^2$ at the point $(1, 0)$ intersect the parabola a second time?

We need, first, to find the equation of the normal line. please note that $y' = 1 - 2x$. That means, (1pt)

$$m_T = y'\Big|_{x=1} = -1 \Rightarrow m_N = 1$$

(1pt) (1pt)

$\Rightarrow$ Equation of the normal line is $y - 0 = 1(x-1)$

$\Rightarrow y = x - 1$ (1pt)

Intersection between $y = x - 1$ & $y = x - x^2$ means

$x - x^2 = x - 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$ (1pt)

If $x = 1 \Rightarrow y = 0$ X

If $x = -1 \Rightarrow y = -2 \Rightarrow$ The point is $(-1, -2)$

(pts) 4. If $g(x) = \dfrac{4 + xf(x)}{\sqrt{x}}$, $f(4) = 2$ and $f'(4) = -2$, then find $g'(4)$.

$$g'(x) = \frac{[f(x) + x f'(x)]\sqrt{x} - \frac{1}{2\sqrt{x}}[4 + x f(x)]}{x}$$

(2pts)

$$\Rightarrow g'(4) = \frac{[f(4) + 4 f'(4)] * 2 - \frac{1}{4}[4 + 4 f(4)]}{4}$$

$$= \frac{[2 - 8] * 2 - \frac{1}{4}[4 + 4 * 2]}{4} = \frac{-12 - 3}{4}$$

$$= \frac{-15}{4}$$

(3 pts)

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5. For what values of x does the graph of $f(x) = e^x \cos x$ have a horizontal tangent?

$$f'(x) = e^x \cos x - e^x \sin x = 0$$

$$\Rightarrow e^x(\cos x - \sin x) = 0$$

$$\Rightarrow \cos x = \sin x$$

$$\Rightarrow x = \frac{\pi}{4} + n\pi; \quad n = 0, \pm 1, \pm 2, \cdots$$

6. Find the following limits:

(a) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta + \tan \theta}$

$$= \lim_{\theta \to 0} \frac{\frac{\sin\theta}{\theta}}{\frac{\theta}{\theta} + \frac{\tan\theta}{\theta}} = \lim_{\theta \to 0} \frac{\frac{\sin\theta}{\theta}}{1 + \frac{\tan\theta}{\theta}}$$

$$= \frac{1}{1+1} = \frac{1}{2}.$$ (2pts)

(5 pts) (b) $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin x \cos x}{x-\frac{\pi}{2}}$

Let $f(x) = \sin x \cos x \Rightarrow f'(\frac{\pi}{2}) = \lim\limits_{x\to\frac{\pi}{2}} \dfrac{f(x)-f(\frac{\pi}{2})}{x-\frac{\pi}{2}}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin x \cos x - 0}{x-\frac{\pi}{2}} = \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin x \cos x}{x-\frac{\pi}{2}}$ (1 pt)

Now, $f'(x) = \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$ (1 pt)

$\Rightarrow f'(\frac{\pi}{2}) = 0 - 1 = -1$ (1 pt)

$\Rightarrow \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin x \cos x}{x-\frac{\pi}{2}} = f'(\frac{\pi}{2}) = -1$

(3 pts) 7. Let $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx+b & \text{if } x > 2 \end{cases}$

Find the values of m and b that make f differentiable everywhere.

f should be continuous. That means (1 pt)

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^-} f(x) = f(2) \Rightarrow \boxed{4 = 2m+b}$ (1 pt)

Also, $f'(x) = \begin{cases} 2x & \text{if } x < 2 \\ m & \text{if } x > 2 \end{cases}$ (1 pt)

f is diff. at $x=2$ if $f'_+(2) = f'_-(2) \Rightarrow 4 = m \Rightarrow$

$4 = 8 + b \Rightarrow b = -4.$ (1 pt)