

King Fahd University of Petroleum and Minerals  
 Department of Mathematics and Statistics  
 Math 101 Section 9      Quiz IV (A) (Term 191)

Name : ..... KEY ..... ID #..... Serial #: .....

1. The sum of the absolute maximum and the absolute minimum values of the function

$$f(x) = 2 \cos x + 2 \cos^2 x, \quad \frac{\pi}{2} \leq x \leq 2\pi$$

is  $f'(x) = -2 \sin x + 4(\cos x)(-\sin x) = 0$

$$\Rightarrow \sin x [-2 - 4 \cos x] = 0$$

a) 0

b)  $\frac{7}{2}$

c)  $\frac{5}{2}$

d) -1

e) 1

$$\Rightarrow \sin x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}$$

$$\downarrow$$

$$x = \pi, 2\pi$$

$$x = \frac{2\pi}{3} \text{ or } \frac{5\pi}{3}$$

$$f\left(\frac{\pi}{2}\right) = 0$$

$$f\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$f\left(\frac{5\pi}{3}\right) = \frac{3}{2}$$

$$\text{Sum} = 4 - \frac{1}{2} = \frac{7}{2}$$

$$f(\pi) = -2 + 2 = 0$$

$$f(2\pi) = 2 + 2 = 4$$

2. The critical numbers of the function  $f(x) = (1 + x + x^2)e^{-x}$  are:

a) 0 and -1

b) 0 only

c) 1 only

d) 0 and 1

e) 1 and -1

$$f'(x) = (1+2x)e^{-x} - (1+x+x^2)e^{-x} = 0$$

$$\Rightarrow e^{-x}[x - x^2] = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1$$

3. The value of  $c$  that satisfies the conclusion of the Mean Value Theorem for the function  $f(x) = 2x^2 + 1$  on the interval  $[1, 3]$  is

a)  $c = 2$

b)  $c = \frac{3}{2}$

c)  $c = \frac{5}{2}$

d)  $c = \frac{5}{4}$

e)  $c = \frac{9}{4}$

$$f'(x) = 4x$$

M.V.T  $\Rightarrow$  There exists  $c \in (1, 3)$  s.t

$$\frac{f(3) - f(1)}{3 - 1} = 4c$$

$$\Rightarrow \frac{19 - 3}{2} = 4c \Rightarrow 4c = 8 \Rightarrow c = 2$$

4. If  $f'(x) \leq 2$  for all  $x$  and  $f(0) = A$ , then the largest possible value of  $f(2)$  is

a)  $A$

b)  $4 - A$

c)  $2 - A$

d)  $2 + A$

e)  $4 + A$

let  $x \in [0, 2]$

$f(x)$  is continuous on  $[0, 2]$  and differentiable on  $(0, 2)$ .

$\therefore$  By MVT, there is at least  $c \in (0, 2)$  such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}, \text{ but } f'(c) \leq 2$$

$$\Rightarrow f'(c) = \frac{f(2) - A}{2} \leq 2 \Rightarrow f(2) \leq 4 + A$$

5. Let  $f(x) = \frac{3}{8}x^{8/3} - \frac{6}{5}x^{5/3} + \frac{3}{2}x^{2/3}$ . If

$C =$  the number of critical points of  $f(x) = 2$

$m =$  the number of local minimum of  $f(x) = 1$

$M =$  the number of local maximum of  $f(x) = 0$

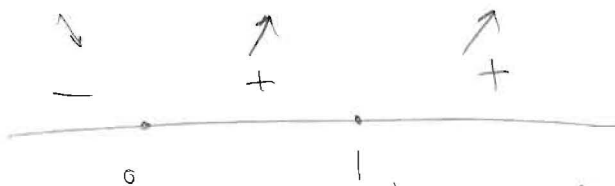
then  $2C + m - M =$

- a) 4
- b) 5
- c) 3
- d) 0
- e) -1

$$f'(x) = x^{5/3} - 2x^{2/3} + x^{-1/3} = 0$$

$$\Rightarrow x^{-1/3} [x^2 - 2x + 1] = 0$$

$$\Rightarrow x^{-1/3} (x-1)^2 = 0$$



$$f'(x) = 0 \Rightarrow x = 1 \in D_f \quad \& \quad f'(x) \text{ DNE} \Rightarrow x = 0 \in D_f$$

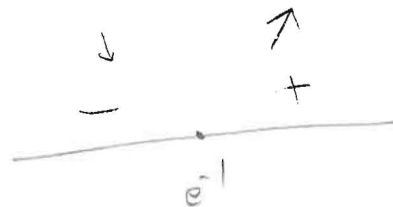
6. Let  $f(x) = x \ln x$ . Then the graph of  $f(x)$  is

- a) decreasing on  $(0, \infty)$
- b) increasing on  $(0, \infty)$
- c) increasing on  $\left(\frac{1}{e}, \infty\right)$  and decreasing on  $\left(0, \frac{1}{e}\right)$
- d) increasing on  $\left(0, \frac{1}{e}\right)$  and decreasing on  $\left(\frac{1}{e}, \infty\right)$
- e) increasing on  $(e, \infty)$  and decreasing on  $(0, e)$

$$f'(x) = 1 + \ln x = 0$$

$$\Rightarrow \ln x = -1$$

$$\Rightarrow x = e^{-1}$$



7. The graph of  $f(x) = \frac{e^x}{e^x + 1}$

- a) is concave up on  $(0, \infty)$
- b) is concave up on  $(-\infty, 0)$
- c) has two inflection points
- d) has no inflection points
- e) is concave up on  $(-\infty, \infty)$

$$f'(x) = \frac{(e^x+1)e^x - e^x(e^x)}{(e^x+1)^2} = \frac{e^x}{(1+e^x)^2}$$

$$\begin{aligned} f''(x) &= \frac{(e^x+1)^2 e^x - e^x \cdot 2(e^x+1)e^x}{(e^x+1)^4} \\ &= \frac{(e^x+1)e^x - 2e^{2x}}{(e^x+1)^3} \\ &= \frac{e^x - e^{2x}}{(e^x+1)^3} = \frac{e^x(1-e^x)}{(e^x+1)^3} \end{aligned}$$

$$f''(x) = 0 \Rightarrow 1 - e^x = 0 \Rightarrow x = 0$$

$f''(x)$  exists for all  $x$

$f''$   $\frac{\text{up}}{+}$   $\frac{\text{down}}{-}$   
 $\uparrow$  inflection pt.

8. If the graph of  $f(x) = \frac{2}{9}x^3 + Ax^2 - \frac{4}{3}x + B + 1$  has a local minimum at the point  $(1, 1)$ , then  $3A + 9B =$

- a) 8
- b) 0
- c) 2
- d) -9
- e) -1

$$f'(x) = \frac{2}{3}x^2 + 2Ax - \frac{4}{3}$$

$$1 = f(1) = \frac{2}{9} + A - \frac{4}{3} + B + 1 \quad \text{--- (1)}$$

$$0 = f'(1) = \frac{2}{3} + 2A - \frac{4}{3} = 0 \Rightarrow 2A = \frac{2}{3} \Rightarrow A = \frac{1}{3}$$

Using (1), we get

$$\frac{2}{9} - 1 + B + 1 = 1 \Rightarrow B = 1 - \frac{2}{9} = \frac{7}{9}$$

$$\Rightarrow 3A + 9B = 1 + 7 = 8$$