King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 9 Quiz IV (A) (Term 191)

Name : KEY ID #: Serial #:

1. The sum of the absolute maximum and the absolute minimum values of the function

$$f(x) = 2\cos x + 2\cos^2 x, \qquad \frac{\pi}{2} \le x \le 2\pi$$

is

$f'(x) = -2\sin x + 4(\cos x)(-\sin x) = 0$

$\Rightarrow \sin x[-2-4\cos x] = 0$

$\Rightarrow \sin x = 0$ & $\cos x = -\frac{1}{2}$

$x = \pi, 2\pi$ $\quad$ $x = \frac{2\pi}{3}$ & $\frac{5\pi}{3}$

$f(\frac{\pi}{2}) = 0$

$f(\pi) = -2+2 = 0$

$f(\frac{2\pi}{3}) = -\frac{1}{2}$

$f(2\pi) = 2+2 = 4$

$f(\frac{5\pi}{3}) = 3/2$

Sum $= 4 - \frac{1}{2} = 7/2$

a) 0
b) $\frac{7}{2}$ ✓
c) $\frac{5}{2}$
d) -1
e) 1

2. The critical numbers of the function $f(x) = (1 + x + x^2)\, e^{-x}$ are:

$f'(x) = (1+2x)e^{-x} - (1+x+x^2)e^{-x} = 0$

$\Rightarrow e^{-x}[x - x^2] = 0$

$\Rightarrow x = 0$ & $x = 1$

a) 0 and -1
b) 0 only
c) 1 only
d) 0 and 1 ✓
e) 1 and -1

3. The value of c that satisfies the conclusion of the **Mean Value Theorem** for the function $f(x) = 2x^2 + 1$ on the interval $[1, 3]$ is

a) $c = 2$

b) $c = \frac{3}{2}$

c) $c = \frac{5}{2}$

d) $c = \frac{5}{4}$

e) $c = \frac{9}{4}$

$f'(x) = 4x$

M.V.T $\Rightarrow$ There exists $c \in (1, 3)$ s.t

$\frac{f(3) - f(1)}{3 - 1} = 4c$

$\Rightarrow \frac{19 - 3}{2} = 4c \Rightarrow 4c = 8 \Rightarrow c = 2$

4. If $f'(x) \leq 2$ for all x and $f(0) = A$, then the largest possible value of $f(2)$ is

let $x \in [0, 2]$

$f(x)$ is continuous on $[0, 2]$ and differentiable on $(0, 2)$.

$\therefore$ By MVT, there is at least $c \in (0, 2)$ such that

a) A

b) $4 - A$

c) $2 - A$

d) $2 + A$

e) $4 + A$

$f'(c) = \frac{f(2) - f(0)}{2 - 0}$, but $f'(c) \leq 2$

$\Rightarrow f'(c) = \frac{f(2) - A}{2} \leq 2 \Rightarrow f(2) \leq 4 + A$

5. Let $f(x) = \frac{3}{8}x^{8/3} - \frac{6}{5}x^{5/3} + \frac{3}{2}x^{2/3}$. If

C = the number of critical points of $f(x)$ = 2

m = the number of local minimum of $f(x)$ = 1

M = the number of local maximum of $f(x)$ = 0

then $2C + m - M =$

$f'(x) = x^{5/3} - 2x^{2/3} + x^{-1/3} = 0$

$\Rightarrow x^{-1/3}[x^2 - 2x + 1) = 0$

$\Rightarrow x^{-1/3}(x-1)^2 = 0$

a) 4
b) 5
c) 3
d) 0
e) −1

$-$ $\searrow$ $+$ $\nearrow$ $+$ $\nearrow$

0 1

$f'(x) = 0 \Rightarrow x = 1 \in D_f$ & $f'(x)$ DNE $\Rightarrow x = 0 \in D_f$

6. Let $f(x) = x \ln x$. Then the graph of $f(x)$ is

$f'(x) = 1 + \ln x = 0$

$\Rightarrow \ln x = -1$

$\Rightarrow x = e^{-1}$

a) decreasing on $(0, \infty)$
b) increasing on $(0, \infty)$
c) increasing on $\left(\frac{1}{e}, \infty\right)$ and decreasing on $\left(0, \frac{1}{e}\right)$
d) increasing on $\left(0, \frac{1}{e}\right)$ and decreasing on $\left(\frac{1}{e}, \infty\right)$
e) increasing on (e, ∞) and decreasing on $(0, e)$

$-$ $\searrow$ $+$ $\nearrow$

e^{-1}

$$f'(x) = \frac{(e^x+1)e^x - e^x(e^x)}{(e^x+1)^2} = \frac{e^x}{(1+e^x)^2}$$

7. The graph of $f(x) = \dfrac{e^x}{e^x+1}$

$$f''(x) = \frac{(e^x+1)^2 e^x - e^x \cdot 2(e^x+1)e^x}{(e^x+1)^4}$$

$$= \frac{(e^x+1)e^x - 2e^{2x}}{(e^x+1)^3}$$

$$= \frac{e^x - e^{2x}}{(e^x+1)^3} = \frac{e^x(1-e^x)}{(e^x+1)^3}$$

a) is concave up on $(0, \infty)$
b) is concave up on $(-\infty, 0)$
c) has two inflection points
d) has no inflection points
e) is concave up on $(-\infty, \infty)$

$f''(x) = 0 \Rightarrow 1 - e^x = 0 \Rightarrow x = 0$

$f''(x)$ exists for all x

f'' up down
$+$ 0 $-$
Inflection pt.

8. If the graph of $f(x) = \frac{2}{9}x^3 + Ax^2 - \frac{4}{3}x + B + 1$ has a local minimum at the point $(1, 1)$, then $3A + 9B =$:

$$f'(x) = \frac{2}{3}x^2 + 2Ax - \frac{4}{3}$$

a) 8
b) 0
c) 2
d) -9
e) -1

$$1 = f(1) = \frac{2}{9} + A - \frac{4}{3} + B + 1 \quad (1)$$

$$0 = f'(1) = \frac{2}{3} + 2A - \frac{4}{3} = 0 \Rightarrow 2A = \frac{2}{3}$$

$$\Rightarrow A = \frac{1}{3}$$

Using (1), we get

$$\frac{2}{9} - 1 + B + 1 = 1 \Rightarrow B = 1 - \frac{2}{9} = \frac{7}{9}$$

$$\Rightarrow 3A + 9B = 1 + 7 = 8$$