King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 9 Quiz IV (B) (Term 191)

Name : KEY ID #.................................. Serial #: ....

1. The absolute maximum and absolute minimum for the function $f(x) = x\sqrt{9-x^2}$ on the interval $[-3, 3]$ are respectively.

$$f'(x) = x \cdot \frac{-2x}{2\sqrt{9-x^2}} + \sqrt{9-x^2} = 0$$

$$\Rightarrow \frac{-x^2 + 9 - x^2}{\sqrt{9-x^2}} = 0 \Rightarrow -2x^2+9=0$$

$$\Rightarrow x^2 = \frac{9}{2} \Rightarrow x = \pm\frac{3}{\sqrt{2}} \in (-3,3)$$

Check $f(-3) = 0$

$f\left(\frac{-3}{\sqrt{2}}\right) = \frac{-3}{\sqrt{2}}\sqrt{9-\frac{9}{2}} = -\frac{9}{2}$ Min.

$f\left(\frac{3}{\sqrt{2}}\right) = \frac{3}{\sqrt{2}} \cdot \frac{3}{\sqrt{2}} = \frac{9}{2}$ Max.

$f(3) = 0$

a) $\frac{9}{2}, 0$
b) $3, -3$
c) $3, 0$
d) $\frac{9}{2}, \frac{-9}{2}$
e) $3, \frac{-9}{2}$

2. The number of critical points of $f(x) = (x - x^3)^{-1/3}$ is

$$f'(x) = -\frac{1}{3}(x-x^3)^{-2/3}(1-3x^2)$$

$f'(x) = 0 \Rightarrow 1 - 3x^2 = 0 \Rightarrow x = \pm\frac{1}{\sqrt{3}} \in D_f$

$f'(x)$ DNE if $x - x^3 = 0 \Rightarrow x(1-x^2) = 0$

$\Rightarrow x = 0, \pm 1 \notin D_f$

a) 3
b) 2
c) 4
d) 5
e) 1

3. If $f(x) = \begin{cases} C & \text{if } \ x = 0 \\ Ae^x & \text{if } \ 0 < x < 1 \\ \ln x + B & \text{if } \ 1 \le x \le 2 \end{cases}$ satisfies

the conditions of the **Mean Value Theorem**, then $A + B - C =$

f is cont. on $[0, 2] \Rightarrow$

cont from right at $x=0 \Rightarrow C = A$

cont. at $x=1 \Rightarrow Ae = B$

$$f'(x) = \begin{cases} Ae^x & 0<x<1 \\ \frac{1}{x} & 1<x<2 \end{cases}$$

$f'_+(1) = f'_-(1) \Rightarrow 1 = Ae \Rightarrow A = \frac{1}{e}$

$\Rightarrow C = \frac{1}{e} \Rightarrow B = 1 \Rightarrow A+B-C = 1$

a) $\frac{1}{e}$

b) e

c) 1

d) $\frac{1}{e^2}$

e) $1+e$

4. If $f(1) = 10$ and $f'(x) \ge 2$ for $1 \le x \le 4$, then a possible choice for $f(4)$ is (**Hint: You may apply the Mean Value Theorem**)

$$f'(c) = \frac{f(4)-f(1)}{4-1} = \frac{f(4)-10}{3} \geq 2$$

$$\Rightarrow f(4) \geq 6 + 10 = 16$$

a) 8

b) 14

c) 12

d) 10

e) 16

5. If $f'(x) = \dfrac{x^2(x-1)}{x+2}$, then which one of the following statement is **TRUE**?

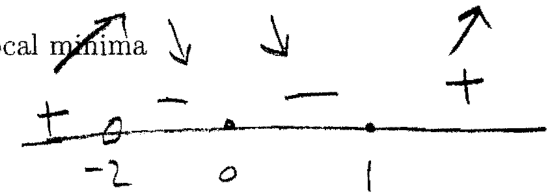
a) f has one local minimum and no local maxima

b) f has one local minimum and one local maximum

c) f is increasing on $(-2, 1)$

d) f is decreasing on $(1, \infty)$

e) f has one local maximum and no local minima



local min. at $x = 1$ only
No local max.

6. The function $g(x) = e^{-\frac{1}{x}}$ is

a) decreasing on $(0, \infty)$

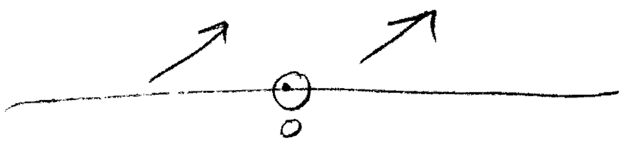
b) increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$

c) increasing on $(-\infty, 0)$ and $(0, \infty)$

d) increasing on $(0, 1)$ and decreasing on $(1, \infty)$

e) decreasing on $(-\infty, 0)$ and $(0, \infty)$

$g'(x) = e^{-\frac{1}{x}} \cdot \frac{+1}{x^2} > 0$ for all $x \neq 0$



7. If (α, β) is the point of inflection of the curve $f(x) = \tan x - 4x$ for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, then $2\alpha - \beta + 2$

$f'(x) = \sec^2 x - 4$

a) 2

b) 0

c) $\frac{\pi}{4}$

d) $-\frac{\pi}{4}$

e) $1 - \pi$

$f''(x) = 2\sec^2 x \tan x$

$f(\alpha) = \beta \Rightarrow \beta = \tan\alpha - 4\alpha$

$f''(\alpha) = 0 \Rightarrow 0 = \underbrace{2\sec^2\alpha}_{\neq 0} \tan\alpha \Rightarrow \tan\alpha = 0 \Rightarrow \alpha = 0$

$\Rightarrow \beta = \tan 0 - 4(0) = 0$

$\Rightarrow 2\alpha - \beta + 2 = 2$

8. The function $f(x) = e^x - x^2$ is

$f'(x) = e^x - 2x$

a) concave-down on $(-\infty, \infty)$

b) concave-up on $(-\infty, 0]$

c) concave-down on $[0, \infty)$

d) concave-up on $[0, \infty)$

e) concave-down on $(-\infty, 0]$

$f''(x) = e^x - 2 = 0$

$\Rightarrow e^x = 2 \Rightarrow x = \ln 2$

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0 $\ln 2$

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