1. Using **three** approximating rectangles and **left endpoints**, the area under the graph of $f(x) = \dfrac{1}{x^3+1}$ from $x = 0$ to $x = 3$ is approximately equal to

When $f(x) = \dfrac{6}{x^3+1}$, The answer is $\dfrac{29}{3}$

(a) $\dfrac{29}{18}$

(b) $\dfrac{10}{27}$

(c) $\dfrac{29}{3}$

(d) $\dfrac{15}{8}$

(e) $\dfrac{20}{9}$

$\Delta x = \dfrac{3-0}{3} = 1$

0 1 2 3

$$A \approx f(0)\,\Delta x + f(1)\,\Delta x + f(2)\,\Delta x$$
$$= 1 \cdot 1 + \tfrac{1}{2} \cdot 1 + \tfrac{1}{9} \cdot 1$$
$$= \frac{3}{2} + \frac{1}{9}$$
$$= \frac{27+2}{18} = \frac{29}{18}$$

2. If $F(x) = \displaystyle\int_x^2 \frac{t}{\ln t}\,dt$, then $F'(x) =$

(a) $-\dfrac{x}{\ln x}$

(b) $\dfrac{x}{\ln x}$

(c) $\dfrac{2}{\ln 2} - \dfrac{x}{\ln x}$

(d) $\dfrac{x}{\ln x} - \dfrac{2}{\ln 2}$

(e) $-\dfrac{2}{\ln 2}$

$$F(x) = -\int_2^x \frac{t}{\ln t}\,dt$$

$$\Rightarrow F'(x) = -\frac{x}{\ln x}, \text{ by FTC}$$

3. $\int \frac{\sqrt{y}-y}{y^3}\,dy = \int y^{-\frac{5}{2}} - y^{-2}\,dy = -\frac{2}{3}y^{-3/2} - \frac{y^{-1}}{-1} + C$

$= \frac{1}{y} - \frac{2}{3}\cdot\frac{1}{y^{3/2}} + C$

$= \frac{1}{y} - \frac{2}{3}\cdot\frac{1}{\sqrt{y^3}} + C$

(a) $\frac{1}{y} - \frac{2}{3}\cdot\frac{1}{\sqrt{y^3}} + C$

(b) $\frac{1}{\sqrt{y^3}} - \frac{1}{y} + C$

(c) $\frac{1}{y^2} - \frac{1}{3}\cdot\frac{1}{\sqrt[3]{y}} + C$

(d) $\frac{2}{y} + \frac{3}{\sqrt{y^3}} + C$

(e) $\frac{3}{\sqrt{y}} - \frac{1}{y} + C$

4. Which one of the following substitutions is **correct**?

✓(a) $\int \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx = \int \cos w\,dw$, where $w = \sqrt{x}$.

(b) $\int x\,e^{x^2+4}\,dx = \int e^w\,dw$, where $w = x^2+4$

(c) $\int \frac{3x^2+4x+1}{x^3+2x^2+2x}\,dx = \int \frac{1}{w}\,dw$, where $w = x^3+2x^2+2x$

(d) $\int \frac{1}{x^2+4}\,dx = \int \frac{1}{w}\,dw$, where $w = x^2+4$

(e) $\int (x^2+4x)(x^3+3x^2)^4\,dx = \int \frac{1}{3}w^4\,dw$, where $w = x^3+3x^2$

5. The **area** of the region bounded by the curves $y = \sqrt{x}$ and $y = \frac{1}{2}x$ is equal to

(a) $\frac{4}{3}$

(b) $\frac{5}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

(e) $\frac{1}{4}$

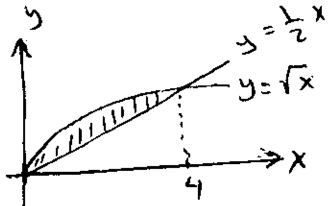
$A = \int_0^4 (\sqrt{x} - \tfrac{1}{2}x)\, dx$

$= \tfrac{2}{3}x^{3/2} - \tfrac{1}{4}x^2 \Big]_0^4$

$= \tfrac{2}{3}4^{3/2} - 4$

$= \tfrac{2}{3}\cdot 8 - 4 = \frac{16}{3} - \frac{12}{3}$

$= \frac{4}{3}$



pts of intersection

$\sqrt{x} = \tfrac{1}{2}x$

$2\sqrt{x} = x$

$4x = x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0,\ x = 4$

6. The velocity (in m/s) of a particle moving along a line is given by

$$v(t) = t^2 + 3t - 4.$$

The **total distance** traveled during the time interval $[0, 2]$ is equal to

(a) $5\,m$

(b) $10\,m$

(c) $\frac{10}{3}\,m$

(d) $\frac{89}{6}\,m$

(e) $\frac{2}{3}\,m$

$d = \int_0^2 |t^2+3t-4|\, dt$: $t^2+3t-4 = (t-1)(t+4) = 0$

$\Rightarrow t = 1, -4$

VH1 $\quad - \quad +$ on $0 \quad 1 \quad 2$

$= \int_0^1 -t^2-3t+4\, dt$

$+ \int_1^2 t^2+3t-4\, dt$

$= -\frac{t^3}{3} - \frac{3}{2}t^2 + 4t\Big]_0^1 + \frac{1}{3}t^3 + \frac{3}{2}t^2 - 4t\Big]_1^2$

$= (-\frac{1}{3} - \frac{3}{2} + 4) - 0 + (\frac{8}{3} + 6 - 8) - (\frac{1}{3} + \frac{3}{2} - 4)$

$= -\frac{1}{3} + \frac{8}{3} - \frac{1}{3} - \frac{3}{2} - \frac{3}{2} + 4 + 6 - 8 + 4$

$= 2 - 3 + 6$

$= 5$

7. Let f be an **odd** and continuous function on $(-\infty, \infty)$.
If $\int_{-1}^{3} f(x)dx = A$, then $\int_{1}^{3} f(x)\,dx =$

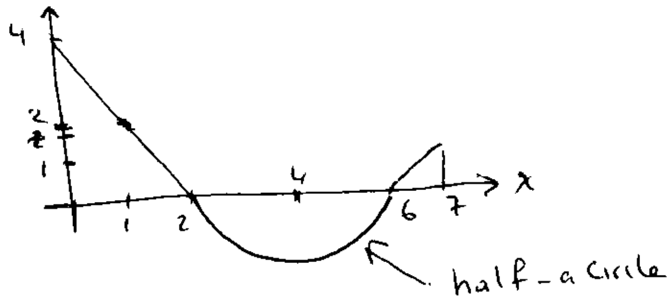
(a) A
(b) $-A$
(c) 0
(d) $2A$
(e) $-3A$

$$\int_{-1}^{3} f(x)\,dx = \int_{-1}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$A = 0 + \int_{1}^{3} f(x)\,dx$$

$$\Longrightarrow \int_{1}^{3} f(x)\,dx = A$$

8. For the function g whose graph is given below, $\int_{1}^{7} g(x)dx =$



(a) $\frac{3}{2} - 2\pi$
(b) $\frac{3}{2} + 2\pi$
(c) $2 - \pi$
(d) $\frac{5}{2} - \pi$
(e) $\frac{1}{2} + 2\pi$

$$\int_{1}^{7} g(x)\,dx = \int_{1}^{2} g(x)\,dx + \int_{2}^{6} g(x)\,dx + \int_{6}^{7} g(x)\,dx$$

$$= \frac{1}{2}\cdot 1\cdot 2 - \frac{1}{2}\pi(2)^2 + \frac{1}{2}\cdot 1\cdot 1$$

$$= 1 - 2\pi + \frac{1}{2}$$

$$= \frac{3}{2} - 2\pi$$

$$\boxed{\int_{0}^{7} g(x)dx = \frac{9}{2} - 2\pi}$$

9. $\int 2^{x^2+\log_2 x}\,dx =$ $\int 2^{x^2} \cdot 2^{\log_2 x}\, dx$

$= \int 2^{x^2} \cdot x\, dx$: $u = x^2 \Rightarrow du = 2x\, dx$

$= \frac{1}{2} \int 2^u\, du$

$= \frac{1}{2} \cdot \frac{2^u}{\ln 2} + C$

$= \frac{2^{x^2}}{2\ln 2} + C$

$= \frac{1}{\ln 4} 2^{x^2} + C$

(a) $\dfrac{1}{\ln 4} \cdot 2^{x^2} + C$

(b) $\ln 2 \cdot 2^{x^2} + C$

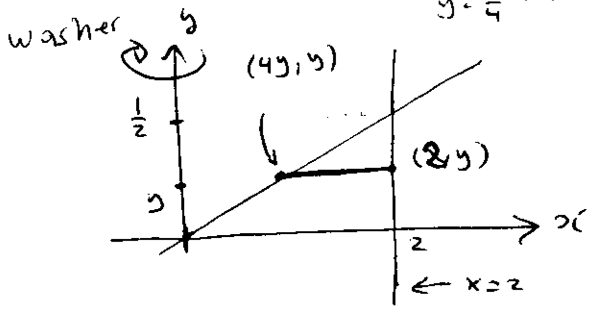
(c) $\dfrac{1}{\ln 2} \cdot 2^{x^2} + C$

(d) $\dfrac{1}{\ln 2} \cdot 2^{x} + C$

(e) $\dfrac{1}{\ln 4} \cdot 2^{x} + C$

10. The **volume** of the solid obtained by rotating the region bounded by the lines $y = \dfrac{x}{4}$, $x = 2$, and $y = 0$ about the y-axis is

$y = \frac{x}{4} \Rightarrow x = 4y$



(a) $\dfrac{4\pi}{3}$

(b) $\dfrac{5\pi}{4}$

(c) $\dfrac{7\pi}{4}$

(d) $\dfrac{5\pi}{8}$

(e) $\dfrac{6\pi}{5}$

$V = \int_0^{1/2} \pi\,(2^2 - (4y)^2)\, dy$

$= \pi \int_0^{1/2} 4 - 16y^2\, dy$

$= \pi \cdot \left. 4y - \frac{16}{3} y^3 \right]_0^{1/2}$

$= \pi \cdot \left(2 - \frac{16}{3} \cdot \frac{1}{8}\right)$

$= \pi \cdot \left(2 - \frac{2}{3}\right)$

$= \frac{4\pi}{3}$

11. Which one of the following statements is **TRUE**:

(f is a continuous function on $(-\infty, \infty)$)

✓ (a) If $f(x) \geq \sin x$ for $0 \leq x \leq \pi$, then $\int_0^{\pi} f(x)dx \geq 2$

(b) $\frac{d}{dx}\left[\int_1^5 f(t)\,dt\right] = f(5) - f(1)$

(c) $\int_2^2 f(x)\,dx = 2$

(d) $\int_{-1}^1 |f(x)|\,dx = 2\int_0^1 |f(x)|\,dx$

(e) If $f(x) < 0$ for $0 \leq x \leq 1$, then $\int_0^1 f(x)\,dx > 0$.

12. $\int_1^{3/2} \frac{3}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx = \int_1^{3/2} \frac{3}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx$

$u = 1 - \frac{1}{x^2} \Rightarrow du = \frac{2}{x^3}\,dx$

$x = 1 \Rightarrow u = 0$

$x = \frac{3}{2} \Rightarrow u = 1 - \frac{4}{9} = \frac{5}{9}$

$= \frac{3}{2}\int_0^{5/9} \sqrt{u}\,du$

$= \frac{3}{2}\cdot\frac{2}{3}\,u^{3/2}\Big]_0^{5/9}$

$= \left(\frac{5}{9}\right)^{3/2} = \frac{5}{9}\sqrt{\frac{5}{9}} = \frac{5\sqrt{5}}{27}$

(a) $\frac{5\sqrt{5}}{27}$

(b) $\frac{5\sqrt{5}}{9}$

(c) $\frac{\sqrt{5}}{3}$

(d) $\frac{2\sqrt{5}}{9}$

(e) $\sqrt{5}$

$$\boxed{\int_1^{3/2} \frac{9}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx = \frac{5\sqrt{5}}{9}}$$

13. $\lim_{n\to\infty} \sum_{i=1}^{n} 6\frac{i^2}{n^3} e^{i^3/n^3} =$ $\lim_{n\to\infty} \sum_{i=1}^{n} 6 \left(\frac{i}{n}\right)^2 e^{(i/n)^3} \cdot \frac{1}{n}$

$\Delta x = \frac{1}{n} \Rightarrow [a,b] = [0,1]$

(a) $2e - 2$

(b) $2e$

(c) $3e - 3$

(d) $3e$

(e) $6e - 1$

$= \int_0^1 6x^2 e^{x^3}\,dx$, $u = x^3 \Rightarrow du = 3x^2\,dx$

$x=0 \Rightarrow u=0$

$x=1 \Rightarrow u=1$

$= 2\int_0^1 e^u\,du$

$= 2 \cdot e^u \Big]_0^1$

$= 2(e-1)$

$= 2e - 2$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} 9 \frac{i^2}{n^3} e^{i^3/n^3} = 3e-3}$$

14. The base of a solid is the region enclosed by the curves $y = x^2$ and $y = 2x - x^2$. If the cross sections perpendicular to the x-axis are **semicircles**, then the volume of the solid is equal to

(a) $\frac{\pi}{60}$

(b) $\frac{\pi}{40}$

(c) $\frac{3\pi}{50}$

(d) $\frac{2\pi}{15}$

(e) $\frac{4\pi}{7}$

pts of intersection:

$x^2 = 2x - x^2$

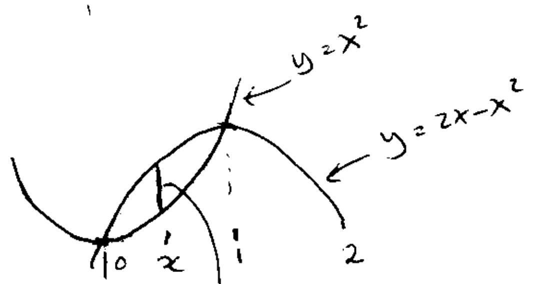
$2x^2 = 2x$

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, \quad x = 1$



d = diameter $= (2x - x^2) - x^2$

$= 2x - 2x^2$

$r = \frac{d}{2} = x - x^2$

$A(x) = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi (x - x^2)^2$

$V = \int_0^1 A(x)\,dx$

$= \frac{\pi}{2} \int_0^1 (x - x^2)^2\,dx$

$= \frac{\pi}{2} \int_0^1 x^2 - 2x^3 + x^4\,dx$

$= \frac{\pi}{2} \cdot \frac{1}{3}x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5 \Big]_0^1$

$= \frac{\pi}{2} \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \frac{\pi}{2} \cdot \frac{10 - 15 + 6}{30} = \frac{\pi}{2} \cdot \frac{1}{30} = \frac{\pi}{60}$

15. If f is a continuous function such that

$$\int_1^{x^2} f(t)\,dt = (x-1)e^{2x} + \int_x^{x^3} e^{-t} f(t)dt,$$

then $f(0) =$

Diff use F.T.C :

$f(x^2)\cdot 2x = (x-1)e^{2x}\cdot 2 + e^{2x}\cdot 1$
$+ e^{-x^3} f(x^3)\cdot 3x^2 - e^{-x} f(x)\cdot 1$

(a) -1
(b) -2
(c) -3
(d) 0
(e) 1

Sub. $x=0$:

$f(0)\cdot 0 = -1\cdot 2 + 1 + 0 - f(0)$

$0 = -2 + 1 - f(0)$

$\Rightarrow f(0) = -1$

$\int_1^{x^2} f(t)\,dt = (x-1)e^{3x} + \int_x^{x^3} e^{-t} f(t)\,dt$

$\Rightarrow f(0) = -2$