King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math 102 Sec# 7, Quiz # 5 (sections 7.4 and 7.8), Semester 191

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Name: Student ID#: SR#

Q1) Express the following function as a sum of partial fractions and **evaluate** the coefficients.

$$f(x) = \frac{x^6}{x^2-4}$$

$$\begin{array}{r|l} & x^4+4x^2+16 \\ \hline x^2-4 & x^6 \\ & -x^6 \pm 4x^4 \\ \hline & 4x^4 \\ & -4x^4 \pm 16x^2 \\ \hline & 16x^2 \\ & -16x^2 \pm 64 \\ \hline & 64 \end{array}$$

$$f(x) = x^4+4x^2+16+\frac{64}{x^2-4}$$

$$= x^4+4x^2+16+\frac{64}{(x-2)(x+2)}$$

$$= x^4+4x^2+16+\frac{A}{x-2}+\frac{B}{x+2}$$

$$= x^4+4x^2+16+\frac{16}{x-2}-\frac{16}{x+2} \quad *$$

Q2) Evaluate the integral or show that it diverges.

$$\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

$$\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx = \lim_{t\to\infty} \int_1^{t} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$

when $x = t \longrightarrow u = \sqrt{t}$

$x = 1 \longrightarrow u = 1$, So the integral becomes

$$\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx = \lim_{t\to\infty} \int_1^{\sqrt{t}} 2e^{-u}\, du$$

$$= \lim_{t\to\infty} -2e^{-u}\Big]_1^{\sqrt{t}}$$

$$= \lim_{t\to\infty} \left[-2e^{-\sqrt{t}} + 2e^{-1}\right]$$

$$= 0 + 2e^{-1}$$

$$= 2e^{-1}. \quad \divideontimes$$

The integral converges to $2e^{-1}$.

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Name: Solution Student ID#: SR#

Q1) Express the following function as a sum of partial fractions and **evaluate** the coefficients.

$$f(x) = \frac{x^5}{x^2 - 4}$$

$$\begin{array}{r|l} & x^3 + 4x \\ \hline x^2-4 & x^5 \\ & -(x^5 - 4x^3) \\ \hline & 4x^3 \\ & -(4x^3 - 16x) \\ \hline & 16x \end{array}$$

$$f(x) = x^3 + 4x + \frac{16x}{x^2-4}$$

$$= x^3 + 4x + \frac{16x}{(x-2)(x+2)}$$

$$= x^3 + 4x + \frac{A}{x-2} + \frac{B}{x+2}$$

$$= x^3 + 4x + \frac{8}{x-2} + \frac{8}{x+2} .$$

Q2) Evaluate the integral or show that it diverges.

$$\int_{-1}^{1} \frac{1+x}{\sqrt[3]{x^4}}\,dx$$

The integrand has an infinite discontinuity at $x=0$.

$$\int_{-1}^{1} \frac{1+x}{\sqrt[3]{x^4}}\,dx = \int_{-1}^{0} \frac{1+x}{\sqrt[3]{x^4}}\,dx + \int_{0}^{1} \frac{1+x}{\sqrt[3]{x^4}}\,dx$$

Let us check the second improper integral

$$\int_{0}^{1} \frac{1+x}{\sqrt[3]{x^4}}\,dx = \lim_{t\to 0^+} \int_{t}^{1} \frac{1+x}{x^{4/3}}\,dx$$

$$= \lim_{t\to 0^+} \int_{t}^{1} \left(x^{-4/3} + x^{-1/3}\right) dx$$

$$= \lim_{t\to 0^+} \left[-3x^{-1/3} + \frac{3}{2}x^{2/3}\right]_t^1$$

$$= \lim_{t\to 0^+} \left[-3 + \frac{3}{2} + \frac{3}{\sqrt[3]{t}} + \frac{3}{2}t^{3/2}\right]$$

since $\lim_{t\to 0^+} \frac{3}{\sqrt[3]{t}} = +\infty$, the original integral is divergent. ※