King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math 102 Sec# 7, Quiz # 7 (sections 11.3, 11.4, 11.5), Semester 191

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Name: Solution Student ID#: SR#

Q1) Determine whether the series converges or diverges.

(a) $\sum_{n=1}^{\infty} n\, e^{-n^2}$ (b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{n^2+2n-1}$

(a) We use the integral test with $f(x) = x e^{-x^2}$ on $[1,\infty)$

(1) clearly f is positive on $[1,\infty)$

(2) clearly f is continuous on $[1,\infty)$

(3) $f'(x) = -2x^2 e^{-x^2} + e^{-x^2} = e^{-x^2}(1-2x^2) < 0$ for $x \geq 1$

Thus f is decreasing. Therefore we compute the integral

$$\int_1^\infty f(x) = \lim_{t\to\infty} \int_1^t x e^{-x^2} dx = \lim_{t\to\infty} -\frac{e^{-x^2}}{2}\Big]_1^t$$

$$= \lim_{t\to\infty} -\frac{e^{-t^2}}{2} + \frac{e^{-1}}{2} = \frac{e^{-1}}{2}$$

Since the integral is convergent, the series is also Conv. $\ast$

(b) We use the L.C.T. with $b_n = \frac{\sqrt{n^2}}{n^2} = \frac{n}{n^2} = \frac{1}{n}$

$$\lim_{n\to\infty} \left|\frac{a_n}{b_n}\right| = \lim_{n\to\infty} \frac{n\sqrt{n^2+4}}{n^2+2n-1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}\sqrt{1+\frac{4}{n^2}}}{\cancel{n^2}\left(1+\frac{2}{n}-\frac{1}{n^2}\right)}$$

$$= 1$$

Since the series $\sum b_n = \sum \frac{1}{n}$ is divergent p-series with $p=1$, the series $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{n^2+2n-1}$ is also

divergent by the L C T.

Q2) Find the smallest number of terms of the following series that we need to add so that |error| < 0.1.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{2n+3}}$$

According to the Estimation Theorem for alternating series

$|error| = R_n \leq b_{n+1} < 0.1$

where $b_{n+1} = \frac{1}{\sqrt{2(n+1)+3}} = \frac{1}{\sqrt{2n+5}}$ hence we need

$$\frac{1}{\sqrt{2n+5}} < \frac{1}{10}$$

$$\sqrt{2n+5} > 10$$

$$2n+5 > 100$$

$$n > \frac{95}{2} = 47.5$$

So take $n \geq 48$.