King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 106

Exam I

191

Monday 07/10/2019

Net Time Allowed: 75 Minutes

MASTER VERSION

1. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, $g'(3) = 5$, then $\left(\frac{f}{f-g}\right)'(3)$ is equal to

(a) 8
(b) 16
(c) 32
(d) 44
(e) 12

2. The slope of the tangent line to the graph of $x^2y^2 + xe^{\frac{y}{2}} = 4$ at the point $(4, 0)$ is equal to

(a) $-\dfrac{1}{2}$
(b) $\dfrac{1}{2}$
(c) $-\dfrac{3}{2}$
(d) $\dfrac{3}{2}$
(e) $-\dfrac{2}{3}$

3. The number of the different equations of the normal lines of slope 1 to the graph of $y = \dfrac{1}{x+1}$ is

(a) 1

(b) 0

(c) 2

(d) 4

(e) 3

4. The value(s) of the constant c that makes the function

$$g(x) = \begin{cases} x^2 - c^2 & \text{if} \quad x < 4 \\ cx + 20 & \text{if} \quad x \geq 4 \end{cases}$$

continuous on $(-\infty, \infty)$ is (are)

(a) -2

(b) 20

(c) 4 and -4

(d) 4

(e) -2 and 2

5. If the line $2x + y = b$ is tangent to the parabola $y = ax^2$ at $x = 2$, then $a - b =$

(a) $-\dfrac{5}{2}$

(b) $\dfrac{3}{2}$

(c) $-\dfrac{3}{2}$

(d) $\dfrac{1}{2}$

(e) $-\dfrac{1}{2}$

6. Let g be a twice differentiable function. If $f(x) = g(\sqrt{x})$, then $f''(x)$ is equal to

(a) $\dfrac{1}{4x}g''(\sqrt{x}) - \dfrac{1}{4\sqrt{x^3}}\,g'(\sqrt{x})$

(b) $\dfrac{1}{4x}\,g''(\sqrt{x})$

(c) $-\dfrac{1}{4\sqrt{x^3}}\,g''(\sqrt{x})$

(d) $\dfrac{1}{2\sqrt{x}}\,g''(\sqrt{x})$

(e) $\dfrac{1}{2\sqrt{x}}\,g''(\sqrt{x}) - \dfrac{1}{4x}$

7. $\lim_{x\to 1.5^-} \dfrac{2x^2-3x}{|2x-3|}$ is equal to

(a) $-\dfrac{3}{2}$

(b) 3

(c) $\dfrac{3}{2}$

(d) 1

(e) -1

8. Which one of the following statements is true about the graph of the function $y = \dfrac{x+1}{x^2-x-2}$?

(a) It has a y intercept and has asymptotes at $x = 2$ and $y = 0$.

(b) It has an $x-$ intercept and one vertical asymptote.

(c) It passes through $y = -\dfrac{1}{2}$ and has asymptotes at $x = 2$ and $x = 0$.

(d) It has two vertical asymptotes and one horizontal asymptote.

(e) It passes through $x = 2$ and has asymptotes at $x = -2$ and $y = 0$.

9. Which of the following statements is true about $f(x) = x^{2/3}$?

(a) f has a vertical tangent line at $(0, 0)$.
(b) f has no vertical tangent line.
(c) f is differentiable on $(-\infty, \infty)$.
(d) f has a horizontal tangent line at $(0, 0)$.
(e) f has a vertical tangent line at $x = 1$.

10. The value of f, g, f', g' are given by the table

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	9	8	5
4	-3	3	2	-6

Then $\frac{d}{dx}(g(x + f(x)))$ at $x = 3$ is

(a) -54
(b) 5
(c) -48
(d) -6
(e) 27

11. The position of a particle is given by the equation of motion $s(t) = \dfrac{t}{t+1}$ where t is measured in seconds and s in meters. Then the average velocity v_{ave} in the time interval $[2, 2+h]$ and the velocity v at $t = 2$ are given by

(a) $v_{ave} = \dfrac{1}{9+3h}\, m/sec$ and $v = \dfrac{1}{9}\, m/sec$

(b) $v_{ave} = \dfrac{12}{18+h}\, m/sec$ and $v = \dfrac{1}{9}\, m/sec$

(c) $v_{ave} = \dfrac{1}{9+5h}\, m/sec$ and $v = \dfrac{1}{9}\, m/sec$

(d) $v_{ave} = \dfrac{4+h}{6+h}\, m/sec$ and $v = \dfrac{2}{3}\, m/sec$

(e) $v_{ave} = \dfrac{1}{18+3h}\, m/sec$ and $v = \dfrac{1}{18}\, m/sec$

12. If $f(x) = [\![x]\!] + [\![-x]\!]$, where $[\![t]\!]$ is the greatest integer less than or equal to t, then the $\lim\limits_{x\to 3} f(x)$

(a) exists and equal to -1.

(b) does not exist because $\lim\limits_{x\to 3} f(x) \neq f(3)$.

(c) exists and is equal to -2.

(d) does not exist because $\lim\limits_{x\to 3} [\![x]\!]$ and $\lim\limits_{x\to 3} [\![-x]\!]$ do not exist.

(e) exists and is equal to 0.

13. Which one of the following functions has a removable discontinuity at $x = 1$?

(a) $f(x) = \begin{cases} x^2 - 1 & \text{if} \quad x < 1 \\ 2x - 2 & \text{if} \quad x > 1 \end{cases}$

(b) $f(x) = \dfrac{1}{(x-1)^2}$

(c) $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if} \quad x \neq 1 \\ 1 & \text{if} \quad x = 1 \end{cases}$

(d) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if} \quad x \neq 1 \\ 2 & \text{if} \quad x = 1 \end{cases}$

(e) $f(x) = \dfrac{|x-1|}{x-1}$

14. If $f(x) = \dfrac{\sqrt{9x^2+1}}{5-2x}$, then

(a) $\lim\limits_{x \to -\infty} f(x) = \dfrac{3}{2}$ and $\lim\limits_{x \to \infty} f(x) = -\dfrac{3}{2}$

(b) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to \infty} f(x) = \dfrac{3}{2}$

(c) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to \infty} f(x) = 0$

(d) $\lim\limits_{x \to -\infty} f(x) = -\dfrac{3}{2}$ and $\lim\limits_{x \to \infty} f(x) = \dfrac{3}{2}$

(e) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to \infty} f(x) = -\dfrac{3}{2}$

15. If $\lim_{x \to 0} \frac{\sqrt{mx+n}-2}{x} = 1$, then $m+n=$

(a) 8

(b) 7

(c) 9

(d) 0

(e) 1