

MATH 201- Sec04-Quiz 3

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1 Exercise 1(5 points)

Find the symmetric equations of the line D of intersection between the two planes

$$x + 2y + 3z = 3, \quad 2x - y + z = 1.$$

Find the intersection of D with the xy -plane

2 Exercise 2(5 points)

Find and sketch the domain of $f(x, y) = \frac{1}{\ln(4x^2 - y + 1)}$. Find the level curves and sketch some of them.

Answer (1):

$$\begin{aligned} \cdot) \quad & \cancel{x+2y+3z=3} \quad x+2y+3z=3 \Rightarrow \vec{n}_1 = \langle 1, 2, 3 \rangle \\ & 2x-y+z=1 \Rightarrow \vec{n}_2 = \langle 2, -1, 1 \rangle \end{aligned}$$

$$\text{Let } \vec{u} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = \langle 5, 5, -5 \rangle. \quad \vec{u} \text{ is parallel}$$

to the line.

It is clear that $(0, 0, 1)$ is a common point.

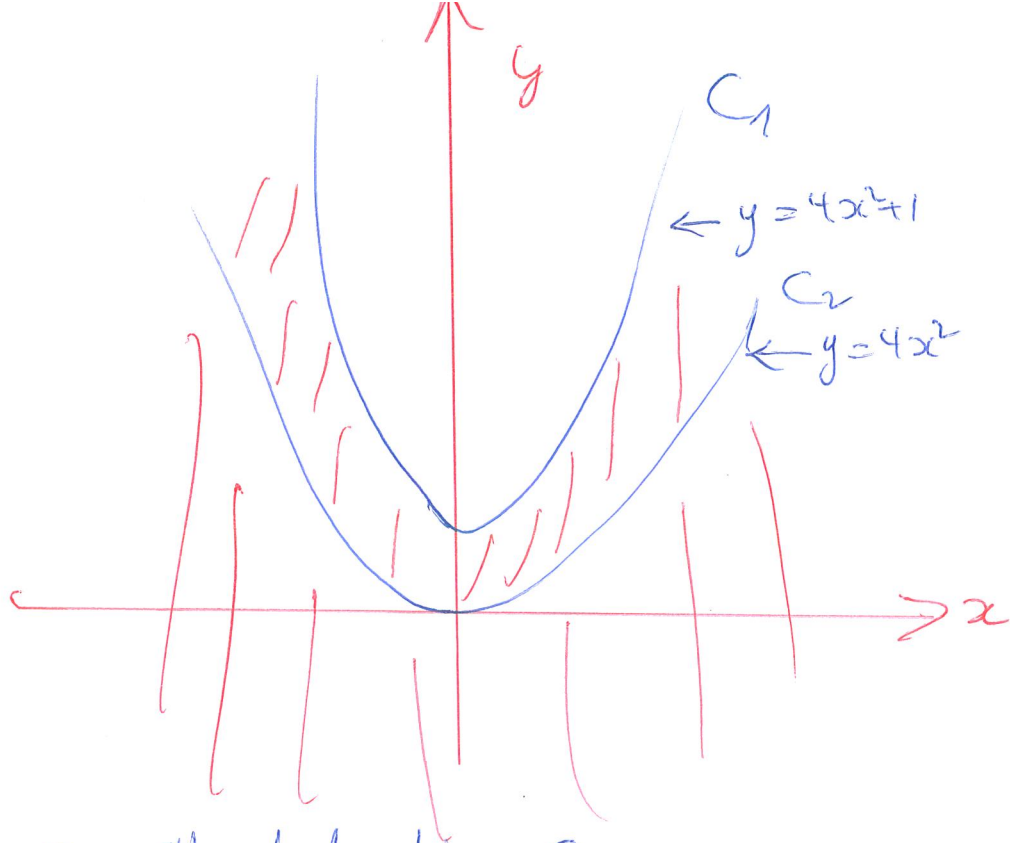
$$\begin{aligned} \text{Then, the symmetric equations are } \quad & \frac{x-0}{5} = \frac{y-0}{5} = \frac{z-1}{-5} \\ \Leftrightarrow \quad & x=y=1-z \end{aligned}$$

$$\cdot) \quad xy\text{-plane} \Rightarrow z=0 \Rightarrow x=y=1-z=1-0=1$$

Conclusion: The intersection point is $(1, 1, 0)$.

Answer 2:

$$\begin{aligned} \frac{1}{\ln(4x^2 - y + 1)} & \text{ exists if and only if } 4x^2 - y + 1 > 0 \text{ and } \ln(4x^2 - y + 1) \neq 0 \\ \Rightarrow & \cancel{4x^2 - y} \quad y < 4x^2 + 1 \text{ and } 4x^2 - y + 1 \neq 1 \\ \Rightarrow & y < 4x^2 + 1 \text{ and } y \neq 4x^2 \\ & C_1: y = 4x^2 + 1 \quad C_2: y = 4x^2 \end{aligned}$$



Under C_1 without touching C_2 .

* Level curves: $C_k = \{(x, y) : f(x, y) = k\}$

$$\Rightarrow \{(x, y) : \frac{1}{\ln(4x^2 - y + 1)} = k\} = \{(x, y) : 4x^2 - y + 1 = e^{\frac{1}{k}}\}$$

$$\Rightarrow \{(x, y) : y = 4x^2 + 1 - e^{\frac{1}{k}}\} \left. \begin{array}{l} \text{parabole} \\ \text{symmetry axis: } y\text{-axis} \\ k \in \mathbb{R} \setminus \{0\} \end{array} \right\}$$

