KFUPM – Department of Mathematics & Statistics
MATH201 – Term 191
Quiz **1A** (Duration: 20 Minutes)

Name: Solution ID #: ______ Section #: ______

Question 1: Sketch the curve and eliminate the parameter to **find** a Cartesian equation of the curve: $x = \sqrt{t+1},\ y = \sqrt{t-3}$

Solution:

$x^2 = t+1 \Rightarrow t = x^2 - 1$

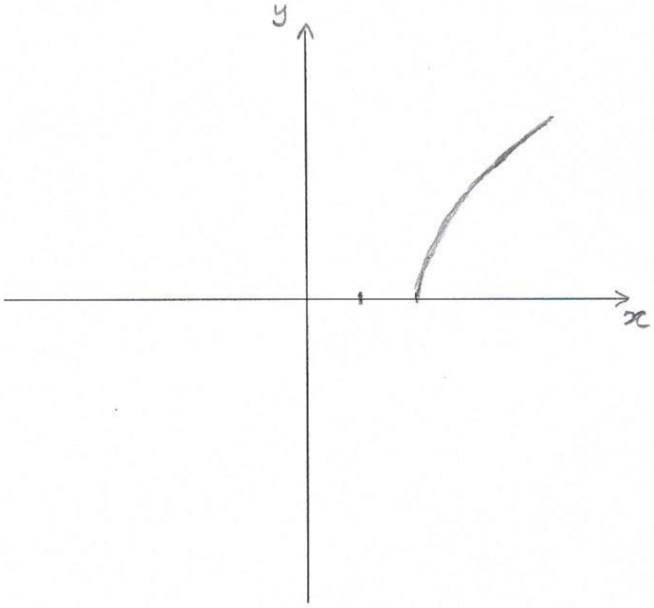
$y^2 = t-3 \Rightarrow t = y^2 + 3$

We obtain:

$$x^2 - 1 = y^2 + 3$$

or

$$x^2 - y^2 = 4, \quad y \geq 0.$$



Question 2: Find the length of the curve $x = e^t - t,\ y = 4e^{t/2},\ 0 \leq t \leq 2.$

Solution: $\frac{dx}{dt} = e^t - 1$, $\frac{dy}{dt} = 2e^{t/2}$.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (e^t-1)^2 + \left(2e^{t/2}\right)^2$$

$$= e^{2t} - 2e^t + 1 + 4e^t$$

$$= e^{2t} + 2e^t + 1 = (e^t+1)^2$$

The Length of the Curve is

$$L = \int_0^2 \sqrt{(e^t+1)^2}\, dt$$

$$= \int_0^2 (e^t+1)\, dt = \left[e^t + t\right]_0^2$$

$$= e^2 + 2 - 1 = e^2 + 1$$

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Quiz **1B** (Duration: 20 Minutes)

Name: Solution ID #: ________ Section #: ____

Question 1: Sketch the curve and eliminate the parameter to **find** a Cartesian equation of the curve: $x = \cos^2 t,\ y = 1 - \sin t,\quad 0 \le t \le \pi/2$

Solution:

$\cos^2 t = x$

and $1 - \sin t = y$

$\sin t = 1 - y$

$\sin^2 t = (1-y)^2$

We have:

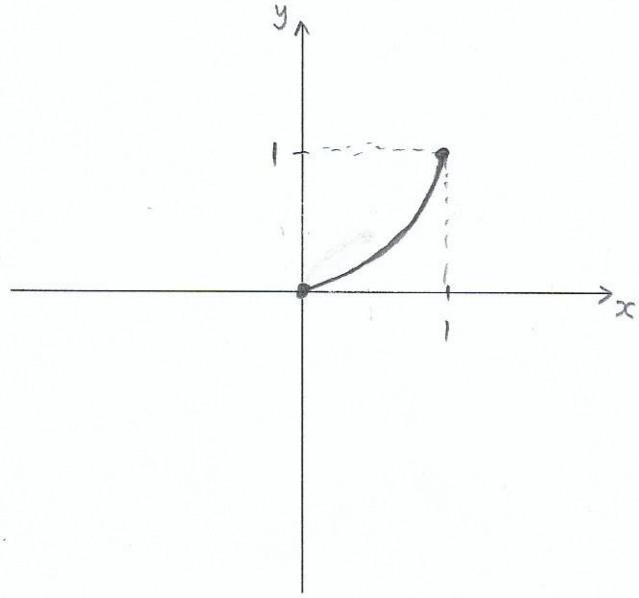
$\cos^2 t + \sin^2 t = 1$

$\Rightarrow x + (1-y)^2 = 1$

$\Rightarrow x + 1 - 2y + y^2 = 1$

$\Rightarrow x = 2y - y^2,$

$0 \le x \le 1$



Question 2: For the curve $x = e^t,\ y = te^{-t}$, find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

Solution:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^{-t} - te^{-t}}{e^t} = (1-t)e^{-2t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left((1-t)e^{-2t}\right)}{dx/dt} = \frac{-e^{-2t} + (1-t)(-2e^{-2t})}{e^t}$$

$$= \frac{-e^{-2t} - 2e^{-2t} + 2te^{-2t}}{e^t}$$

$$= (2t-3)e^{-3t}.$$

$$\boxed{\frac{d^2y}{dx^2} = 0 \text{ if } t = \frac{3}{2}.}$$

$\frac{d^2y}{dx^2}$ − − − − − − (3/2) + + + + + +

The curve is concave upward when $t > \frac{3}{2}$.

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Quiz **1C** (Duration: 20 Minutes)

Name: Solution ID #: ____________ Section #: ______

Question 1: Sketch the curve and eliminate the parameter to **find** a Cartesian equation of the curve: $x = \sin^2 t,\ y = 1 - \cos t,\quad 0 \le t \le \pi$

Solution:

$\sin^2 t = x$

and $\cos t = 1 - y$

$\cos^2 t = (1-y)^2$

Since $\sin^2 t + \cos^2 t = 1$,

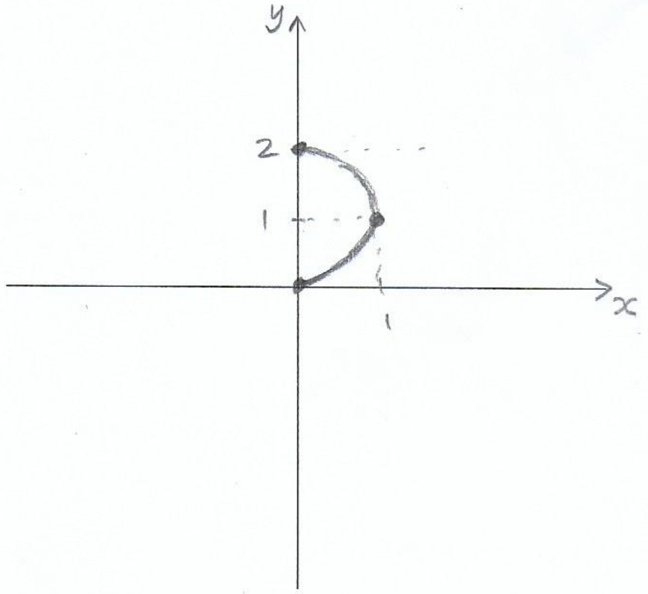
we obtain

$x + (1-y)^2 = 1$

$\Rightarrow \quad x + 1 - 2y + y^2 = 1$

$\Rightarrow \quad x = 2y - y^2,$

$0 \le x \le 1$



Question 2: Find the points on the curve $x = t^3 - 3t,\ y = t^3 - 3t^2$, where the tangent is horizontal or vertical.

Solution:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 6t}{3t^2 - 3} = \frac{t^2 - 2t}{t^2 - 1}$$

Horizontal Tangent: $dy/dt = 0, \quad \frac{dx}{dt} \neq 0$

$t^2 - 2t = 0$

$t(t-2) = 0 \Rightarrow t = 0$ or $t = 2$

$t = 0 \Rightarrow x = 0,\ y = 0$

$t = 2 \Rightarrow x = 2,\ y = -4$

The curve has Horizontal tangent at $(0,0)$ and $(2,-4)$

Vertical tangent: $dx/dt = 0, \quad \frac{dy}{dt} \neq 0$

$t^2 - 1 = 0 \Rightarrow t = \pm 1$

$t = 1 \Rightarrow x = -2,\ y = -2$

$t = -1 \Rightarrow x = 2,\ y = -4$

The curve has Vertical tangent at $(-2,-2)$ and $(2,-4)$