KFUPM – Department of Mathematics & Statistics

MATH201 – Term 191

Quiz 4A (Duration: 25 Minutes)

Name: Solution ID #: ________ Section#: ____

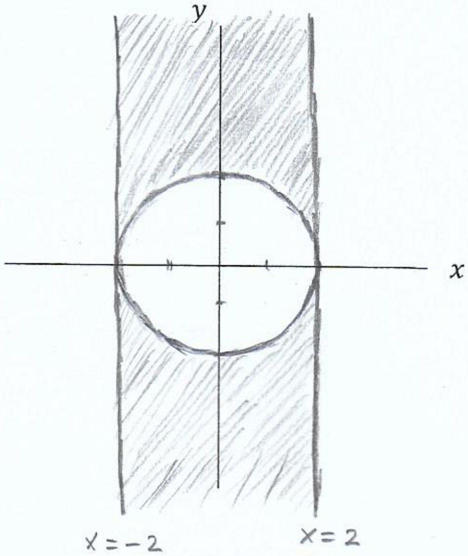
Q1: ***Find*** and ***sketch*** the ***domain*** of the function $f(x,y)=\sqrt{4-x^2}+\sqrt{x^2+y^2-4}$. What is the ***range*** of f?

The domain is

$$D=\{(x,y)\mid x^2\le 4,\ x^2+y^2\ge 4\}$$
$$=\{(x,y)\mid -2\le x\le 2,\ x^2+y^2\ge 4\}$$

The range is $[0,\infty)$

$$0\le\sqrt{4-x^2}+\sqrt{x^2+y^2-4}<\infty$$
$$0\le f(x,y)<\infty$$



Q2: ***Find*** the limit if it exists or ***show*** that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2\cos y}{x^2+2y^2}$

- Along the x-axis ($y=0$),

$$\lim_{(x,y)\to(0,0)}\frac{x^2\cos y}{x^2+2y^2}=\lim_{x\to 0}\frac{x^2}{x^2}=1$$

- Along the y-axis ($x=0$)

$$\lim_{(x,y)\to(0,0)}\frac{x^2\cos y}{x^2+2y^2}=\lim_{y\to 0}\frac{0}{2y^2}=0$$

So, the limit DNE

(b) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2-y^2}{\sqrt{x^2+y^2}}$

using polar coordinates, we obtain:

$$\lim_{(x,y)\to(0,0)}\frac{x^2-y^2}{\sqrt{x^2+y^2}}=\lim_{r\to 0}\frac{r^2(\cos^2\theta-\sin^2\theta)}{r}=\lim_{r\to 0} r\cos 2\theta=0$$

(since $-1\le\cos 2\theta\le 1$)

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Quiz 4B (Duration: 25 Minutes)

Name: Solution ID #: ______ Section#: ______

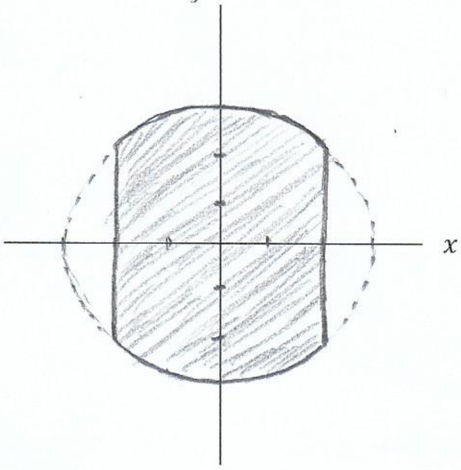
Q1: ***Find*** and ***sketch*** the domain of the function $f(x,y) = \sqrt{4-x^2} + \sqrt{9-x^2-y^2}$. What is the range of f?

The domain is

$$D = \{(x,y) \mid x^2 \le 4,\ x^2+y^2 \le 9\}$$
$$= \{(x,y) \mid -2 \le x \le 2,\ x^2+y^2 \le 9\}$$

The range is $[0,5]$

$$0 \le \sqrt{4-x^2} + \sqrt{9-x^2-y^2} \le 2+3$$
$$0 \le f(x,y) \le 5$$



Q2: ***Find*** the limit if it exists or ***show*** that the limit does not exist.

(a) $\lim\limits_{(x,y)\to(1,0)} \dfrac{xy-y}{(x-1)^2+y^2}$ (b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\sin^2 y}{x^2+2y^2}$

(a)

① Along the x-axis ($y=0$),

$$\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2} = \lim_{(x,y)\to(1,0)} \frac{0}{(x-1)^2} = 0$$

② Along the line $y = x-1$,

$$\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2} = \lim_{x\to1} \frac{(x-1)^2}{2(x-1)^2} = \frac{1}{2}$$

So, the limit DNE

(b)

$$0 \le \frac{x^2}{x^2+2y^2} \le 1$$

$$0 \le \frac{x^2\sin^2 y}{x^2+2y^2} \le \sin^2 y$$

$$0 \le \lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2} \le 0$$

By the Squeeze theorem,

$$\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2} = 0$$

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Quiz 4C (Duration: 25 Minutes)

Name: Solution ID #: Section#:

Q1: ***Find*** and ***sketch*** the ***domain*** of the function $f(x,y) = \sqrt{1-y^2} + \ln(4 - x^2 - y^2)$. What is the ***range*** of f?

The domain is

$$D = \{(x,y) \mid y^2 \le 1,\ x^2+y^2 < 4\}$$

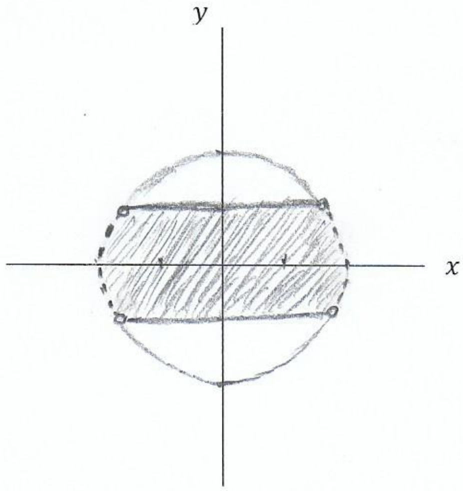
$$= \{(x,y) \mid -1 \le y \le 1,\ x^2+y^2<4\}$$

The range is

$$(-\infty,\ 1+\ln 4]$$

$$-\infty < \sqrt{1-y^2} + \ln(4-x^2-y^2) \le 1+\ln 4$$

$$-\infty < f(x,y) < 1+\ln 4$$



Q2: ***Find*** the limit if it exists or ***show*** that the limit does not exist.

$(a)\ \lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2}$ $(b)\ \lim_{(x,y)\to(0,0)} \frac{y^2\sin^2 x}{2x^2+y^2}$

(a)

⊙ Along the x-axis $(y=0)$

$$\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2} = \lim_{x\to 0} \frac{0}{x^4} = 0$$

⊙ Along the curve $y=x^2$,

$$\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2} = \lim_{x\to 0} \frac{x^4e^{x^2}}{5x^4} = \frac{1}{5}$$

So, the limit DNE

(b)

$$0 \le \frac{y^2}{2x^2+y^2} \le 1$$

$$0 \le \frac{y^2\sin^2 x}{2x^2+y^2} \le \sin^2 x$$

$$0 \le \lim_{(x,y)\to(0,0)} \frac{y^2\sin^2 x}{2x^2+y^2} \le 0$$

By the squeeze theorem,

$$\lim_{(x,y)\to(0,0)} \frac{y^2\sin^2 x}{2x^2+y^2} = 0$$

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Quiz 4D (Duration: 25 Minutes)

Name: Solution ID #: ____________ Section#: ______

Q1: ***Find*** and ***sketch*** the ***domain*** of the function $f(x,y) = \sqrt{4-y^2} + \ln(x^2+y^2-9)$. What is the ***range*** of f?

The domain is

$$D = \{(x,y) \mid y^2 \le 4,\ x^2+y^2 > 9\}$$

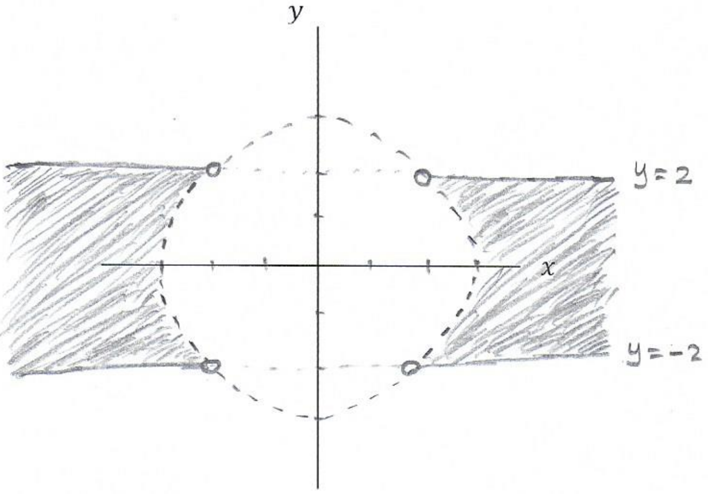
$$= \{(x,y) \mid -2 \le y \le 2,\ x^2+y^2 > 9\}$$

The range is

$$(-\infty, \infty)$$

$$-\infty < \sqrt{4-y^2} + \ln(x^2+y^2-9) < \infty$$

$$-\infty < f(x,y) < \infty$$



Q2: ***Find*** the limit if it exists or ***show*** that the limit does not exist.

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy\cos x}{x^2+2y^2}$

① Along the x-axis $(y=0)$,

$$\lim_{(x,y)\to(0,0)} \frac{xy\cos x}{x^2+2y^2} = \lim_{x\to 0} \frac{0}{x^2} = 0$$

① Along the line $y=x$

$$\lim_{(x,y)\to(0,0)} \frac{xy\cos x}{x^2+2y^2} = \lim_{x\to 0} \frac{x^2\cos x}{3x^2} = \frac{1}{3}$$

So, the limit DNE

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \left(\sqrt{x^2+y^2}\,\ln(x^2+y^2)\right)$

using polar coordinates, we obtain,

$$\lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2}\,\ln(x^2+y^2) = \lim_{r\to 0} r\ln r^2$$

$$= \lim_{r\to 0} \frac{\ln r^2}{1/r}$$

$$= \lim_{r\to 0} \left(\frac{2r/r^2}{-1/r^2}\right) \quad [\text{L'Hospital rule}]$$

$$= \lim_{r\to 0} (-2r)$$

$$= 0$$