MATH 201 – **Quiz#2** (Term 191) **Section** 30

Name: ID:

Exercise 1(5 points)

1) Use vectors to decide wether the triangle with vertices $P(1,-3,2)$, $Q(2,0-4)$ and $R(6,-2,-5)$ is right-angled.

2) Find the values of x such that the angle between the vectors $(2,1,-1)$ and$(1,x,0)$ is $45°$.

3) Find the scalar and vector projections of b onto a, where $a=(4,3)$ and $b=(6,7)$

Answer

1°/ $\vec{PR} = (5,1,-7)$; $\vec{PQ} = (1,3,-6)$; $\vec{QR} = (4,-2,-1)$.

$\vec{PR}\cdot\vec{PQ} \neq 0$; $\vec{PQ}\cdot\vec{QR} \neq 0$; $\vec{PR}\cdot\vec{QR} \neq 0$.

then the triangle is <u>not</u> right-angled.

2°// $(2,1,-1)\cdot(1,x,0) = \sqrt{4+1+1}\cdot\sqrt{1+x^2}\cdot\cos(45^0)$

$= \sqrt{6}\cdot\sqrt{1+x^2}\cdot\sqrt{2}/2 = 2+x$.

$\Leftrightarrow \boxed{x = 1 \pm \sqrt{6}/2}$.

3°// $\cancel{|a| = \sqrt{4^2+3^2}}$ $a=(4,3)$; $b(6,7)$.

$|a| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$.

$\text{Comp}_a(b) = \dfrac{a\cdot b}{|a|} = \dfrac{4\times 6 + 3\times 7}{5} = \dfrac{45}{5} = \boxed{9}$.

$\text{proj}_a(b) = \left(\dfrac{a\cdot b}{|a|}\right)\cdot\dfrac{a}{|a|}$

$= 9\cdot\dfrac{(4,3)}{5} = \boxed{\dfrac{9}{5}\cdot(4,3)} = \left(\dfrac{36}{5}, \dfrac{27}{5}\right)$.

Exercice 2 (5 points)

1) Find two unit vectors orthogonal to both $(3, 2, 1)$ and $(-1, 1, 0)$.

2) Find the area of the parallelogram with vertices $A(-3, 0)$, $B(-1, 3)$, $C(5, 2)$ and $D(3, -1)$.

Answer

1°// It is clear that $(3,2,1) \times (-1,1,0)$ is orthogonal to both a and b.

$$(3,2,1) \times (-1,1,0) = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix} = -i - 1j + 5k$$

$$v = (-1,-1,5).$$

$$|v| = \sqrt{1+1+25} = \sqrt{27} = 3\sqrt{3}$$

$$u = \frac{1}{3\sqrt{3}}(-1,-1,5) \text{ and } -u = -\frac{1}{3\sqrt{3}}(-1,-1,5)$$

are 2 unit vect.

2°// The area of parall. is $|\vec{AB} \times \vec{AD}| = 16.$