MATH 201 – Quiz#5 (Term 191) **Section 30**

Name: ID:

Exercise 1(5 points)
Evaluate the iterated integral by converting to polar coordinates :
a) $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}.dy.dx$
b) $\int_0^{1/2} \int_{\sqrt{3}y}^{\sqrt{1-y^2}} xy^2 dx dy$

Answer

a// $x = r\cos\theta$; $y = r\sin\theta$

$D = \{(x,y) / 0 \le x \le 2 ;\ 0 \le y \le \sqrt{4-x^2}\}$

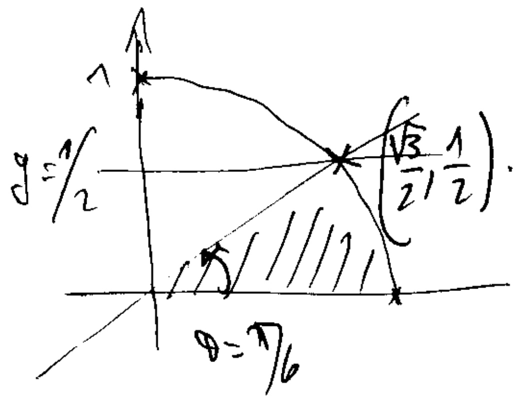
$= \{(r,\theta) / 0 \le r \le 2 ;\ 0 \le \theta \le \pi/2\}.$

2.5

$I = \int_0^{\pi/2} \int_0^2 r e^{-r^2} . dr\, d\theta = \pi/2 \left[-\frac{e^{-r^2}}{2} \right]_0^2 = \pi/2 \left(\frac{1 - e^{-4}}{2} \right)$

$= \pi/4 \left(1 - e^{-4}\right).$

b// The intersection between $x = \sqrt{3}y$ and $x^2 + y^2 = 1$ is $(\sqrt{3}/2, 1/2)$.



$E = \{(r,\theta) / 0 \le r \le 1 ;\ 0 \le \theta \le \pi/6\}.$

$I = \int_0^1 \int_0^{\pi/6} r\cos\theta\, r^2 \sin^2(\theta)\, r\, dr\, d\theta = \left[\frac{r^5}{5}\right]_0^1 \times \left[\frac{\sin^3(\theta)}{3}\right]_0^{\pi/6}$

2.5

$I = \frac{1}{120}$

Exercice 2 (5 points)
Use triple integrals to find the volume of the given solid.
a) The solid enclosed by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$.
b) The solid enclosed by the cylinder $y = x^2$ and and the planes $z = 0$ and $y + z = 1$
Answer

a) The paraboloids intersect when $x^2+z^2 = 8-x^2-z^2$
$\Leftrightarrow$ $x^2+z^2=4$, thus the intersection is the circle $x^2+z^2=4$ and $y=4$. The projection of E onto xz-plane is $x^2+z^2 \leq 4$.

2,5

$$E = \{(x,y,z) / x^2+z^2 \leq y \leq 8-x^2-z^2;\ x^2+z^2 \leq 4\}.$$

Let $D = \{(x,z) / x^2+z^2 \leq 4\}$. By using polar coordinates

$$V = \iiint_E dV = \iint_D \left(\int_{x^2+z^2}^{8-x^2-z^2} dy \right) dA = \iint_D (8-2x^2-2z^2)\, dA$$

$$= \int_0^{2\pi}\int_0^2 (8-2r^2)\, r\, dr\, d\theta = \boxed{16\pi}$$

b) The plane $y+z=1$ intersects xy-plane in line $y=1$, so

$$E = \{(x,y,z) / -1 \leq x \leq 1;\ x^2 \leq y \leq 1;\ 0 \leq z \leq 1-y\}$$

2,5

$$V = \iiint_E dV = \int_{-1}^{1}\int_{x^2}^{1}\int_0^{1-y} dz\, dy\, dx = 8/15$$