

MATH 202.5 (Term 191)

Quiz 1 (Sects. 2.2 & 2.3)

Duration: 20min

Name: _____

ID number: _____

1.) (5pts) Solve the DE: $(y-1) \frac{dy}{dx} = y(y+1) \cos^2 x$.

2.) (5pts) Solve the IVP: $\begin{cases} \frac{dy}{dx} + \frac{x}{x^2-1}y = \frac{\sqrt{x^2-1}}{x^3}, \\ y(2) = 0. \end{cases}$

1.) $y=0$ and $y=-1$ are constant solutions

let $y \neq 0, -1$.

$$\frac{y-1}{y(y+1)} dy = \cos^2 x dx$$

$$\int \left(\frac{2}{y+1} - \frac{1}{y} \right) dy = \int \frac{1 + \cos 2x}{2} dx$$

$$2 \ln|y+1| - \ln|y| = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\ln \left| \frac{(y+1)^2}{y} \right| = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\boxed{\frac{(y+1)^2}{y} = c e^{\frac{x}{2} + \frac{\sin 2x}{4}}}$$

2.) $\int \frac{x}{x^2-1} dx = \ln \sqrt{x^2-1}$
 $e = e = \sqrt{x^2-1}, x > 1$

$$\frac{d}{dx} (y \sqrt{x^2-1}) = \frac{x^2-1}{x^3} = \frac{1}{x} - \frac{1}{x^3}$$

$$y \sqrt{x^2-1} = \int \left(\frac{1}{x} - \frac{1}{x^3} \right) dx$$

$$y = \frac{\ln x + \frac{1}{2x^2} + C}{\sqrt{x^2-1}}$$

$$0 = \frac{\ln 2 + \frac{1}{8} + C}{\sqrt{3}}$$

$$C = -\left(\ln 2 + \frac{1}{8} \right)$$

$$\Rightarrow y = \frac{\ln \left(\frac{x}{2} \right) + \frac{1}{2} \frac{1}{x^2} - \frac{1}{8}}{\sqrt{x^2-1}}, x > 1$$