

Name: _____

ID number: _____

1.) (5pts) Solve the DE: $ye^{-xy} + x \cos 2x + xe^{-xy} \frac{dy}{dx} = 0$.

2.) (5pts) Solve the DE: $\frac{3}{4}x^2 \frac{dy}{dx} + y = y^{1/4}$.

1) $M(x,y) = ye^{-xy} + x \cos 2x +$

$N(x,y) = xe^{-xy}$

$M_y = e^{-xy} - yx e^{-xy}$
 $N_x = e^{-xy} - xy e^{-xy}$) DE is exact

$\begin{cases} \frac{\partial f}{\partial x} = ye^{-xy} + x \cos 2x & (1) \\ \frac{\partial f}{\partial y} = xe^{-xy} & (2) \end{cases}$

(1) $f(x,y) = -e^{-xy} + g(x)$

(2) $\Rightarrow ye^{-xy} + g'(x) = ye^{-xy} + x \cos 2x$

$g'(x) = x \cos 2x$

$g(x) = \int x \cos 2x dx$

integration by parts

$= \frac{x}{2} \sin 2x + \frac{\cos 2x}{4}$

$-e^{-xy} + \frac{x}{2} \sin 2x + \frac{\cos 2x}{4} = C$

implicit solution

2.) Bernoulli DE

$u = y^{1-1/4} = y^{3/4}$

$y = u^{4/3} \Rightarrow \frac{dy}{dx} = \frac{4}{3} u^{1/3} \frac{du}{dx}$

$x^2 \frac{1}{3} \frac{du}{dx} + u^{4/3} = u^{1/3}$

$x^2 \frac{du}{dx} + u = 1$

$\frac{du}{dx} + \frac{1}{x^2} u = \frac{1}{x^2}, x > 0$

$e^{\int \frac{dx}{x^2}} = e^{-1/x}$

$\Rightarrow \frac{d}{dx} (u e^{-1/x}) = \frac{1}{x^2} e^{-1/x}$

$u e^{-1/x} = \int \frac{1}{x^2} e^{-1/x} dx$

$= e^{-1/x} + C$

$u = 1 + C e^{1/x}$

$y^{3/4} = 1 + C e^{1/x}, x \in (0, \infty)$

$y = (1 + C e^{1/x})^{4/3}, x \in (0, \infty)$