MATH 202.5 (Term 162)

Quiz 4 (Sects. 4.6 & 4.7) Duration: 20min

Name: **ID number:**

1.)(5pts)Solve the DE: $y'' - \frac{1}{4}y = \frac{1}{e^{-x}+1}$.

2.)(5pts)Solve the DE: $x^3y''' + 2xy' - 2y = 0$.

---

1) $y'' - \frac{1}{4}y = 0$

Auxiliary equation: $m^2 - \frac{1}{4} = 0$

$m = -\frac{1}{2}, \; m = \frac{1}{2}$

$y_c = c_1 \underbrace{e^{\frac{-x}{2}}}_{y_1} + c_2 \underbrace{e^{\frac{x}{2}}}_{y_2}, \quad x \in (-\infty, \infty).$

$W = \begin{vmatrix} e^{-x/2} & e^{x/2} \\ \frac{-e^{-x/2}}{2} & \frac{1}{2}e^{x/2} \end{vmatrix} = 1$

$y_p = u_1 y_1 + u_2 y_2,$

$u_1' = \frac{-y_2 f(x)}{W} = \frac{-e^{\frac{3x}{2}}}{1+e^x}$

$u_1 = -\int \frac{e^{\frac{3x}{2}}}{1+e^x}\,dx, \quad u = e^{x/2}, \; du = \frac{1}{2}e^{\frac{x}{2}}dx$

$= -2\int \left(1 - \frac{1}{u^2+1}\right) du$

$= -2\left(e^{x/2} - \tan^{-1} e^{x/2}\right)$

$u_2' = \frac{y_1 f(x)}{W} = \frac{e^{x/2}}{1+e^x}$

$u_2 = \int \frac{e^{x/2}}{1+e^x}\,dx, \quad u = e^{x/2}$

$= \int \frac{2}{1+u^2}\,du = 2\tan^{-1} e^{x/2}$

$\Rightarrow y_p = -2 + 2\left(e^{x/2} + e^{-x/2}\right)\tan^{-1} e^{x/2}$

$= -2 + 4\cosh\frac{x}{2}\tan^{-1} e^{x/2}$

$y = y_c + y_p, \quad x \in (-\infty, \infty)$

2.) This is a Cauchy-Euler DE

$y = x^m. \Rightarrow$

$m(m-1)(m-2) + 2m - 2 = 0$

$(m-1)(m^2 - 2m + 2) = 0$

$m = 1, \quad m = 1+i, \quad m = 1-i$

$\Rightarrow y = c_1 x + c_2 x\cos\ln x + c_3 x\sin\ln x,$

$x \in (0, \infty).$