

MATH 202.9 (Term 191)
 Quiz 1 (Sects. 2.2 & 2.3)

Duration: 20min

Name: _____

ID number: _____

- 1.) (5pts) Solve the DE: $(y+1) \frac{dy}{dx} = y(y-1) \sin^2 x$.
 2.) (5pts) Solve the IVP: $\begin{cases} \frac{dy}{dx} + \frac{x}{x^2+1} y = \frac{\sqrt{x^2+1}}{x^3}, \\ y(1) = 0. \end{cases}$

1.) $y=0$ and $y=1$ are constant solutions

let $y \neq 0, 1$

$$\frac{y+1}{y(y-1)} dy = \sin^2 x dx$$

$$\int \left(\frac{2}{y-1} - \frac{1}{y} \right) dy = \int \frac{1 - \cos 2x}{2} dx$$

$$2 \ln|y-1| - \ln|y| = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\ln \left| \frac{(y-1)^2}{y} \right| = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\boxed{\frac{(y-1)^2}{y} = C e^{\frac{x}{2} - \frac{\sin 4x}{4}}}$$

2.) $e^{\int \frac{x}{x^2+1} dx} = e^{\frac{1}{2} \ln(x^2+1)} = \sqrt{x^2+1}$

$$\frac{d}{dx} (y \sqrt{x^2+1}) = \frac{x^2+1}{x^3} = \frac{1}{x} + \frac{1}{x^3}$$

$$y \sqrt{x^2+1} = \int \left(\frac{1}{x} + \frac{1}{x^3} \right) dx$$

$$= \ln|x| - \frac{1}{2} \frac{1}{x^2} + C, x > 0$$

$$y(1) = 0$$

$$\Rightarrow 0 = -\frac{1}{2} + C, C = \frac{1}{2}$$

$$\boxed{y = \frac{\ln x - \frac{1}{2} \frac{1}{x^2} + \frac{1}{2}}{\sqrt{x^2+1}}, x > 0}$$