MATH 202.9 (Term 191)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: ID number:

1.) (5pts) Solve the DE: $3x^2y - x\sin x - y^2e^x = (2ye^x - x^3)\frac{dy}{dx}$.

2.) (5pts) Solve the DE: $(e^{2x}+1)\frac{dy}{dx} + 2(e^{2x}+1)y = y^{1/2}$.

1) $M(x,y) = 3x^2y - x\sin x - y^2e^x$

$N(x,y) = x^3 - 2ye^x$

$\left.\begin{array}{l} M_y = 3x^2 - 2ye^x \\ N_x = 3x^2 - 2ye^x \end{array}\right\}$ DE is exact.

$\begin{cases} \frac{\partial f}{\partial x} = 3x^2y - x\sin x - y^2e^x & (1) \\ \frac{\partial f}{\partial y} = x^3 - 2ye^x & (2) \end{cases}$

(2) $\Rightarrow f(x,y) = x^3y - y^2e^x + g(x)$

(1) $\Rightarrow 3x^2y - y^2e^x + g'(x) = 3x^2y - x\sin x - y^2e^x$

$\Rightarrow g'(x) = -x\sin$

$g(x) = \underbrace{-\int x\sin x\,dx}_{\text{integration by parts}}$

$= x\cos x - \sin x$

$f(x,y) = x^3y - y^2e^x + x\cos x - \sin$

$\boxed{x^3y - y^2e^x + x\cos x - \sin x = C}$

implicit solution.

2) This is Bernoulli's DE

$u = y^{1-\frac{1}{2}} = y^{\frac{1}{2}}$

$y = u^2 \Rightarrow \frac{dy}{dx} = 2u\frac{du}{dx}$

$2u\frac{du}{dx} + 2u^2 = \frac{u}{e^{2x}+1}$

$\frac{du}{dx} + u = \frac{1}{2(e^{2x}+1)}$

e^x is the integrating factor

$\frac{d}{dx}(ue^x) = \frac{e^x}{2(e^{2x}+1)}$

$ue^x = \underbrace{\int \frac{e^x}{2(e^{2x}+1)}\,dx}_{\text{by substitution } v=e^x}$

$ue^x = \frac{1}{2}\tan^{-1}(e^x) + C$

$y^{\frac{1}{2}} = \frac{e^{-x}}{2}\tan^{-1}(e^x) + Ce^{-x}$

$\boxed{y = \left(\frac{e^{-x}}{2}\tan^{-1}(e^x) + Ce^{-x}\right)^2, \; x \in (-\infty, \infty)}$