MATH 202.9 (Term 191)

Quiz 3 (Sects. 4.1.3, 4.2 & 4.3) Duration: 20min

Name: **ID number:**

1.)(3pts) Let L be a linear differential operator. If $Ly_{p_1} = e^x$ and $Ly_{p_2} = e^{-x}$, then find a particular solution of the DE $Ly = e^{x-2} - 2\sinh x$.
2.)(4pts)Use reduction of order to find a second solution y_2 of the DE: $(1-x^2)y'' - 2xy' + 2y = 0$, $x > 1$, giving that $y_1 = x$ is a solution.
3.)(3pts)Solve the DE: $3y''' - 5y'' - 3y' + 5y = 0$.

1.) $e^{x-2} - 2\sinh x = e^{-2}e^{x} - 2\frac{(e^{x} - e^{-x})}{2}$

$= (e^{-2} - 1)e^{x} + e^{-x}$

$\Rightarrow y_p = (e^{-2}-1)y_{p_1} + y_{p_2}$

2.) $y'' - \frac{2x}{1-x^2}y' + \frac{2}{1-x^2}y = 0, \quad x > 1$

$$y_2 = y_1 \int \frac{e^{\int \frac{2x}{1-x^2}dx}}{y_1^2}\,dx$$

$$= x \int \underbrace{\frac{1}{x^2(1-x^2)}}_{\frac{1}{x^2} + \frac{1}{1-x^2}}\,dx$$

$$\frac{1}{x^2} + \frac{\frac{1}{2}}{1-x} + \frac{1/2}{1+x}$$

$$y_2 = -x\left[-\frac{1}{x} - \frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x|\right]$$

$$= -x\left[-\frac{1}{x} + \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\right.$$

$$y_2 = 1 - \frac{x}{2}\ln\left(\frac{x+1}{x-1}\right)$$

3.) The auxiliary equation is

$3m^3 - 5m^2 - 3m + 5 = 0$

$m = 1$ is a root

$$\begin{array}{l|l} 3m^3 - 5m^2 - 3m + 5 & m-1 \\ \hline -\ (3m^3 - 3m^2) & 3m^2 - 2m - 5 \\ \quad -2m^2 - 3m + 5 & \\ -\ (-2m^2 + 2m) & \\ \quad\quad -5m + 5 & \\ \quad\quad -5m + 5 & \\ \quad\quad\quad 0 & \end{array}$$

$3m^2 - 2m - 5 = 0$

$\Delta = 4 + 60 = 64 = 8^2$

$m_1 = \frac{2-8}{6} = -1$

$m_2 = \frac{2+8}{6} = \frac{5}{3}$

$\Rightarrow y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{\frac{5}{3}x}$

MATH 202.5 (Term 191)

Quiz 3 (Sects. 4.1.3, 4.2 & 4.3) Duration: 20min

Name: **ID number:**

1.)(3pts) Let L be a linear differential operator. If $Ly_{p_1} = e^x$ and $Ly_{p_2} = e^{-x}$, then find a particular solution of the DE $Ly = e^{x+2} + 2\cosh x$.

2.)(4pts) Use reduction of order to find a second solution y_2 of the DE: $(1-x^2)y'' + 2xy' - 2y = 0$, giving that $y_1 = 1 + x^2$ is a solution.

3.)(3pts) Solve the DE: $2y''' - 3y'' - 2y' + 3y = 0$.

1.) $e^{x+2} + 2\cosh x = e^2 e^x + 2\left(\frac{e^x + e^{-x}}{2}\right)$

$= (e^2+1)e^x + e^{-x}$

$\Rightarrow y_p = (e^2+1)y_{p_1} + y_{p_2}$

2.) $y'' + \frac{2x}{1-x^2}y' - \frac{2}{1-x^2}y = 0$

$$y_2 = y_1 \int \frac{e^{\int \frac{-2x}{1-x^2}dx}}{y_1^2} dx$$

$$= (1+x^2)\int \underbrace{\frac{1-x^2}{(1+x^2)^2}}_{\frac{1+x^2-2x^2}{(1+x^2)^2}} dx \quad , \quad x \in (-1,1)$$

$\frac{1}{1+x^2} - \frac{2x^2}{(1+x^2)^2}$

$$y_2 = (1+x^2)\left[\tan^{-1}x - \underbrace{\int \frac{2x^2}{(1+x^2)^2}dx}\right]$$

$u' = \frac{-2x}{1+x^2} \qquad u = \frac{1}{1+x^2}$

$v = x \qquad v' = 1$

$$y_2 = (1+x^2)\left[\tan^{-1}x + \frac{x}{1+x^2} - \underbrace{\int \frac{1}{1+x^2}dx}_{\tan^{-1}x}\right]$$

$y_2 = x$

3.) its auxiliary equation is

$2m^3 - 3m^2 - 2m + 3 = 0$

$m = 1$ is a root.

$$\begin{array}{l|l} 2m^3 - 3m^2 - 2m + 3 & m-1 \\ -(2m^3 - 2m^2) & 2m^2 - m - 3 \\ \hline -m^2 - 2m + 3 & \\ -(-m^2 + m) & \\ \hline -3m + 3 & \\ -3m + 3 & \\ \hline 0 & \end{array}$$

$2m^2 - m - 3 = 0$

$\Delta = 1 + 24 = 25 = 5^2$

$m_1 = \frac{1-5}{4} = -1$

$m_2 = \frac{1+5}{4} = \frac{3}{2}$

$\Rightarrow y = c_1 e^x + c_2 e^{-x} + c_3 e^{\frac{3}{2}x}$