MATH 202.5 (Term 191)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 20min

Name: ID number:

1.)(5pts)Find 2 powers series solutions of the DE: $(1-x^2)y''-y=0$.

2.)(5pts)Find a power series solution $y=\sum_{n=0}^{\infty} c_n x^{n-\frac{1}{3}}$ of the DE $3xy''+4y'-y=0$.

1.) $y=\sum_{n=0}^{\infty} c_n x^n, \quad |x|<1$

$(1-x^2)\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} - \sum_{n=0}^{\infty} c_n x^n = 0$

$\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} - \sum_{n=2}^{\infty} c_n n(n-1)x^n - \sum_{n=0}^{\infty} c_n x^n = 0$

$\sum_{k=0}^{\infty} c_{k+2}(k+1)(k+2)x^k - \sum_{k=2}^{\infty} c_k k(k-1)x^k - \sum_{k=0}^{\infty} c_k x^k = 0$

$2c_2+6c_3x-(c_0+c_1x)$

$+\sum_{k=2}^{\infty}\left[c_{k+2}(k+1)(k+2) - c_k(1+k(k-1))\right]x^k = 0$

$$\begin{cases} 2c_2-c_0=0 & \Rightarrow c_2=\frac{c_0}{2} \\ 6c_3-c_1=0 & \Rightarrow c_3=\frac{c_1}{6} \\ c_{k+2}=\frac{1+k(k-1)}{(k+1)(k+2)}c_k, & k=2,3,\dots \end{cases}$$

$c_4=\frac{3}{12}c_2=\frac{c_0}{8}$

$c_5=\frac{7}{20}c_3=\frac{7}{120}c_1$

$y=c_0+c_1x+c_2x^2+c_3x^3+c_4x^4+c_5x^5+\cdots$

$=c_0+c_1x+\frac{c_0}{2}x^2+\frac{c_1}{6}x^3+\frac{c_0}{8}x^4+\frac{7}{120}c_1x^5+\cdots$

$=c_0\underbrace{\left(1+\frac{x^2}{2}+\frac{x^4}{8}+\cdots\right)}_{y_1}+c_1\underbrace{\left(x+\frac{x^3}{6}+\frac{7}{120}x^5+\cdots\right)}_{y_2}$

$y=c_1y_1+c_2y_2$

2.) $3x\sum_{n=0}^{\infty} c_n(n-\frac{1}{3})(n-\frac{4}{3})x^{n-\frac{7}{3}} + 4\sum_{n=0}^{\infty} c_n(n-\frac{1}{3})x^{n-\frac{4}{3}} - \sum_{n=0}^{\infty} c_n x^{n-\frac{1}{3}} = 0$

$x^{-\frac{1}{3}}\left(3\sum_{n=0}^{\infty} c_n(n-\frac{1}{3})(n-\frac{4}{3})x^{n-1} + 4\sum_{n=0}^{\infty} c_n(n-\frac{1}{3})x^{n-1} - \sum_{n=0}^{\infty} c_n x^n\right)=0$

$\frac{4}{3}c_0x^{-1}-\frac{4}{3}c_0x^{-1}+\sum_{n=1}^{\infty} 3c_n(n-\frac{1}{3})(n-\frac{4}{3})x^{n-1}$

$+\sum_{n=1}^{\infty} 4c_n(n-\frac{1}{3})x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$

$\sum_{k=0}^{\infty} 3c_{k+1}(k+\frac{2}{3})(k-\frac{1}{3})x^k + \sum_{k=0}^{\infty} 4c_{k+1}(k+\frac{2}{3})x^k - \sum_{k=0}^{\infty} c_k x^k = 0$

$\sum_{k=0}^{\infty}\left[c_{k+1}(k+\frac{2}{3})(3k+3)\cancel{x^k} - c_k\right]x^k = 0$

$c_{k+1}=\frac{c_k}{(3k+2)(k+1)}, \quad k=0,1,2,\dots$

$c_1=\frac{c_0}{2}$

$c_2=\frac{c_1}{10}=\frac{c_0}{20}$

$y=x^{-\frac{1}{3}}\left(c_0+\frac{c_0}{2}x+\frac{c_0}{20}x^2+\cdots\right)$

$=c_0\underbrace{x^{-\frac{1}{3}}\left(1+\frac{x}{2}+\frac{x^2}{20}+\cdots\right)}_{y_1}$

MATH 202.9 (Term 191)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 20min

Name: ID number:

1.)(5pts)Find 2 powers series solutions of the DE: $(1-x)y''-y'=0$.

2.)(5pts)Find a power series solution $y=\sum_{n=0}^{\infty} c_n x^{n-\frac{1}{2}}$ of the DE $2xy''+3y'-y=0$.

1) $y=\sum_{n=0}^{\infty} c_n x^n$, $|x|<1$

$$(1-x)\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} - \sum_{n=1}^{\infty} c_n n x^{n-1} = 0$$

$$\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} - \sum_{n=2}^{\infty} c_n n(n-1)x^{n-1} - \sum_{n=1}^{\infty} c_n n x^{n-1} = 0$$

$$\sum_{k=0}^{\infty} c_{k+2}(k+1)(k+2)x^k - \sum_{k=1}^{\infty} c_{k+1}k(k+1)x^k - \sum_{k=0}^{\infty} c_{k+1}(k+1)x^k = 0$$

$$2c_2 - c_1 + \sum_{k=1}^{\infty}\left[c_{k+2}(k+1)(k+2) - c_{k+1}(k+1)^2\right]x^k = 0$$

$$\begin{cases} 2c_2 - c_1 = 0 \Rightarrow c_2 = \frac{c_1}{2} \\ c_{k+2} = c_{k+1}\frac{k+1}{k+2}, \quad k=1,2,3,\dots \end{cases}$$

$$c_3 = \frac{2}{3}c_2 = \frac{c_1}{3}$$

$$c_4 = \frac{3}{4}c_3 = \frac{c_1}{4}$$

$$y = c_0 + c_1 x + \frac{c_1}{2}x^2 + \frac{c_1}{3}x^3 + \frac{c_1}{4}x^4 + \dots$$

$$= c_0\underbrace{(1)}_{y_1} + c_1\underbrace{\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)}_{y_2}$$

$$y = c_1 y_1 + c_2 y_2$$

2) $$2x\sum_{n=0}^{\infty} c_n\left(n-\tfrac{1}{2}\right)\left(n-\tfrac{3}{2}\right)x^{n-\frac{5}{2}} + 3\sum_{n=0}^{\infty} c_n\left(n-\tfrac{1}{2}\right)x^{n-\frac{3}{2}} - \sum_{n=0}^{\infty} c_n x^{n-\frac{1}{2}} = 0$$

$$x^{-\frac{1}{2}}\left(\sum_{n=0}^{\infty} 2c_n\left(n-\tfrac{1}{2}\right)\left(n-\tfrac{3}{2}\right)x^{n-1} + \sum_{n=0}^{\infty} 3c_n\left(n-\tfrac{1}{2}\right)x^{n-1} - \sum_{n=0}^{\infty} c_n x^n\right) = 0$$

$$\frac{3}{2}c_0 x^{-1} - \frac{3}{2}c_0 x^{-1} + \sum_{n=1}^{\infty} 2c_n\left(n-\tfrac{1}{2}\right)\left(n-\tfrac{3}{2}\right)x^{n-1} + \sum_{n=1}^{\infty} 3c_n\left(n-\tfrac{1}{2}\right)x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{k=0}^{\infty} 2c_{k+1}\left(k+\tfrac{1}{2}\right)\left(k-\tfrac{1}{2}\right)x^k + \sum_{k=0}^{\infty} 3c_{k+1}\left(k+\tfrac{1}{2}\right)x^k - \sum_{k=0}^{\infty} c_k x^k = 0$$

$$\sum_{k=0}^{\infty}\left[c_{k+1}\left(k+\tfrac{1}{2}\right)(2k+2) - c_k\right]x^k = 0$$

$$c_{k+1} = \frac{c_k}{(2k+1)(k+1)}, \quad k=0,1,2,\dots$$

$$c_1 = c_0$$

$$c_2 = \frac{c_1}{6} = \frac{c_0}{6}$$

$$y = x^{-\frac{1}{2}}\left(c_0 + c_0 x + \frac{c_0}{6}x^2 + \dots\right)$$

$$= c_0\underbrace{x^{-\frac{1}{2}}\left(1 + x + \frac{x^2}{6} + \dots\right)}_{y_1}$$