1. [15 points] An annuity-immediate pays 50 for 5 years, then decreases 5 for 9 years. Find the present value of the annuity.
2. [15 points] A perpetuity-immediate makes quarterly payments and earns annual nominal interest rate of 8% convertible quarterly. The first 10 payments are 20. Starting with the 11th payment all subsequent payments are 15. Find the presents value of of this perpetuity.
3. [15 points] A 20-year annuity immediate with annual payments earns 4%. The first payment is 200 and each subsequent payment is increased by 2% over the previous one. Find the present value of this annuity.
4. [15 points] Smith pays P at the end of each year into a fund that earns an annual rate of i. The accumulated value at the end of 10 years will be double the accumulated value at the end of 5 years. Find i.
5. [15 points] Payments are made to an account at a continuous rate of (4k+kt), where $0<t<10$. Interest is credited at a force of interest $(4+t)^{-1}$. After 10, years the accumulated amount is 10,000. Find k.
6. [15 points] Smith makes payments of P into a fund at the beginning of each year for 10 years with annual effective interest rate 4%. The interest earned at the end of each year reinvested at an annual effective interest rate of 8%. At the end of 10 years, the accumulated value of the 10 payments and the reinvested interest is 10,000. Calculate P.
7. [10 points] John purchases an annuity that makes payments at the beginning of each year for 10 years. The first 5 payments are 1000 and the last are 2000. The annuity earns 4% annually. Find the cost of the annuity.

Q1

$$50a_{\overline{5}|0.05} + \left(\overset{P}{\overset{||}{45}}a_{\overline{9}|0.05} - \overset{Q}{\overset{||}{5}}\left(\frac{a_{\overline{9}|0.05} - 9v^9}{0.05}\right)\right)v^5$$

5.

$$216.47 + \left(319.85 - 5\left(\frac{7.10 - 9(1.05)^{-9}}{0.05}\right)\right)1.05^{-5}$$

$$216.47 + 190.1.05^{-5} = 365.33.$$

Q2

$$20a_{\overline{10}|0.02} + 15a_{\overline{\infty}|0.02}v^{10}$$

$$= 179.65 + 15\frac{1}{0.02}(1.02)^{-10}$$

$$= 794.91$$

Q3

$$200v + 200v^2 1.02 + 200v^3 1.02^2 + \ldots + 200 1.02^{19} v^{20}$$

$$= 200v\left(1 + v1.02 + v^2 1.02^2 + \ldots + 1.02^{19}v^{19}\right)$$

$$= 200v\,\frac{(1.02v)^{20} - 1}{1.02v - 1} = \frac{-0.32182}{-0.01923}\,192.30769$$

$$= 3,218.32$$

Q4.

$$2 s_{\overline{5}|i} = s_{\overline{10}|i}$$

$$2\frac{(1+i)^5 - 1}{i} = \frac{(1+i)^{10} - 1}{i}$$

$$t = (1+i)^5 \qquad 2t - 2 = t^2 - 1$$

$$t^2 - 2t + 1 = 0$$

$$t = 1 \qquad i = 0\%$$

Q5

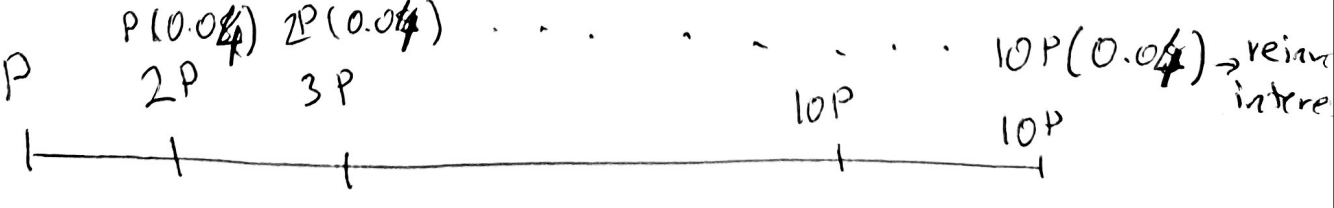
Accumulation factor growth from t to 10

$$e^{\int_t^{10} \frac{1}{4+t}\, dt} = \frac{14}{4+t}$$

Total accumulation $\int_0^{10} k(4+t)\frac{14}{4+t}\, dt = 140k = 10{,}000$

$$k = 71.42.$$

Q6



$$AV = 10P + 0.04P\,(Is)_{\overline{10}|0.08} = 10{,}000$$

$$= 10P + 0.04P\,\frac{s_{\overline{10}|0.08} - 10}{} = 10{,}000 \qquad P = 779.86.$$

Q7 PV = cost

$$1000\,\ddot{a}_{\overline{5}|0.04} + v^5 2000\,\ddot{a}_{\overline{5}|0.04} = 12,240.78$$