Semester 051
Session 2005-2006

Department of Mathematical Sciences
King Fahd University of Petroleum & Mineral, Dhahran

Instructor Dr. A.H.Siddiqi **Course Math-513**

Mathematical Methods for Engineers

Course Description:
Laplace transforms including the convolution theorem, error and gamma functions. The method of Frobenius for series solutions to differential equations. Fourier series, Fourier-Bessel series and boundary value problems, Sturm-Liouville theory. Partial differential equations: separation of variable and Laplace transform and Fourier integrals methods. The heat equation. Laplace equation, and wave equation. Eigenvalue problems for matrices, diagonalization.

Main Objective
The main aim of this course is to introduce certain methods of mathematics which are vital importance for understanding of the problems encountered by engineers.

Book Recommended
Fourier Series and Boundary value problems by James Ward Brown and Ruel V. Churchil, McGraw-Hill International Edition, 6^{th} Edition. 2001.

For Laplace Transforms and Eigenvalue Problems etc WebCt stuff is referred.

Weekly Breakup

Lecture Number	Dates	Content to be Covered	Book/Webct
1	10-9-2005	Definition & Fundamental Properties of the Laplace Transform	Webct Stuff (Chapter 9)
2	12-9-2005	Continued	Section 9.1
3	17-9-2005	Solved Examples	9.1
4	19-9-2005	Inverse of Laplace Transform	Section 9.2 Webct
5	26-9-2005	Shifting Theorems and Derivatives of the Laplace Transform	Section 9.3

6	1-10-2005	Transform of Derivatives and Integrals	Section 9.4.1
7	3-10-2005	Convolution	Section 9.4.2
8	8-10-2005	Unit Impulse and Dirac Delta Function	Section 9.4.3
9	10-10-2005	Application to Differential equations and Integral Equations	Section 9.5
10	15-10-2005	Continuation of the above theme	Section 9.5
11	17-10-2005	The method of Frobenius for series solutions to Differential Equation	Chapter 6 (Webct) Section 6.3
12	24-10-2005	Bessels Equation, Legendres Equation	Section 6.4
26^{th} Oct to 11^{th} Nov.2005 Eit-Ul-Fitr Holiday.			
13	12-11-2005 & 14-11-2005	Legendres Polynomials & Bessel's function, Orthogonal functions and orthogonal series expansions	Section 6.6, 6.5 & 6.4 (See also Chapter 8 of book)
14	17-11-2005	Fourier Bessel's series, Sturm-Liouville Theory	Chapter 8 of the Book p.285, Chapter 6 section 6.7 of Webct
15	19-11-2005	Eigenvalue problems of Matrices	Section 8.3 of the Webct

16	21-11-2005	Diagonalization	Notes to be distributed
17	26-11-2005	Introduction to Partial Differential Equations and Boundary value Problems	Section 11.1 & 11.2 of Chapter 11 of Webct
18	28-11-2005	Continuation	Section 11.3 of Webct
19	03-12-2005	Partial Differential Equations (PDEs) as models of Real World Problems: Heat Equation, Wave Equation, Laplace Equation, Transport Equation, Traffic Equation, Helmholtz Equation, Klein Gordon Equation, Telegraph Equation, Schroedinger Equation, Korteweg de Vries Equation, Euler Equation, Navier Stokes Equation, Maxwell Equations	Webct Chapter 12. Section 12.1 See also pages 4,21,121,130,149. 185,197,264,270 & 275 of the book
20	5-12-2005	Elements of Fourier Analysis for PDE's.	Webct Chapter 12. Section 12.2. See also Chapter 3,4,5 of the Book Prescribed
21	10-12-2005	Method of Separation of Variables for Solving PDE's and Applications to Heat Equation and Wave Equation	Section 1.3 of Webct
22	12-12-2005	Above theme to be continued	Also pages 41 to 47 of the book.
23	17-12-2005	Solution of PDE with initial and boundary conditions: Wave equations, Heat equation , Laplace equation, Black- Scholes Equation	Section 12.4.1,12.4.2 and 12.4..3 of Webct see also chapter 10 of the prescribed book
24	19-12-2005	Continued	〃

25	24-12-2005	Continued	〃
26	31-12-2005	Continued	〃
27	02-01-2006	Revision	〃
5-20 Eid-Al-Adha Break			

Note: Midterm Examination: Monday, 5 Dec 2005, 06:30 to 08:00p.m

Venue: Bldg. 5/103

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Home Work.

Find the Laplace transform of the following functions

1 $f(t) = 4$

2. $f(t) = t^{50}$

3. $f(t) = \cos t$

4. $f(t) = e^{100t}$

5 $f(t) = e^{t} \sin\ 5t$

6. $f(t) = \dfrac{e^{5t} - e^{-6t}}{t}$

7. $f(t) = 1,\ \text{if } 0 \le t < \frac{1}{2}$
$= -1,\ \text{if } \frac{1}{2} \le t < 1$
$= 0,\ \text{otherwise}$

8 Check whether $f(t) = \sin t$ is of exponential order or not?

9 Find $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+2}\right\}$, that is, the inverse of Laplace Transform of $\dfrac{1}{s^2+2}$

10 Apply the first Shifting theorem to find $\mathcal{L}\left\{e^{2t}\sin t\right\}$

11 Using the Laplace transform of f^1 find $\mathcal{L}\left\{\sin kt\right\}$ where $f(t) = \sin kt$.

12 Prove that the Laplace transform of the convolution of two function f and g is equal to the product of the Laplace transform of f and the Laplace transform of g.

Apply Laplace transform to solve the following initial-value problems

13. $y' - 4y = 1, y(0) = 1$

14. $y'' + 4y = e^{-t}, y(0) = 2, y'(0) = 1.$

15. $ty'' + (4t - 2)y' - 4y = 0, y(0) = 1.$

16. Apply the method of Frobenius to obtain two linearly independent series solution of the Differential Equation:

$$2xy'' - y' + 2y = 0$$

about a regular singular point $x = 0$ of the differential equation.

17. Find a particular solution of equations

(i) $(1 - x^2)y'' - 2xy' = 0$

(ii) $(1 - x^2)y'' - 2xy' + 6y = 0$

18. Show that the set $\{1, \cos x, \cos 2x, ...\}$ is orthogonal on the interval $[-\pi, \pi]$

19. Discuss Solution of (a) Periodic Sturm-Liouville problem:

$y'' + \lambda y = 0, y(-1) = y(1), y'(-1) = y'(1)$
on an $[-1,1]$ for cases (i) λ=0 (ii)$\lambda < 0$ (iii) $\lambda > 0$.

(b) $y'' + \lambda y = 0; y(0) = 0, y'(0) = 0.$

20. Solve the problem $y'' + \lambda y(x) = 0; y(-\pi) = y(\pi), y'(-\pi) = y'(\pi).$

Find eigenvalues and eigenvector corresponding to each eigenvalue of the following metrices.

21. $A = \begin{pmatrix} -5 & 0 \\ 1 & 2 \end{pmatrix}$

22. $A = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$

23. $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

24. $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

25. $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{pmatrix}$

For each of the following matrices, produce a matrix that diagonalizes the given matrix, or show that this matrix is not diagonalizable.

26. $A = \begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$

27. $A = \begin{pmatrix} -5 & 3 \\ 0 & 9 \end{pmatrix}$

28. $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$

29. $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{pmatrix}$

30. Write down the examples of partial differential equations of order 1.

31. Write down the examples of partial differential equations of order 2.

32. Explain the concept of hyperbolic, parabolic and elliptic partial differential equations. Give example at latest one of each type of equation.

33. Show the function $u(x-y)=e^{-y}(\sin x+\cos x)$ is a solution of the following problem:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ x>0, y>0$$

subject to the two boundary conditions

$$u(0,y)=\frac{\partial^2 u}{\partial y^2}\Big|_{x=0}, y>0$$
$$u(x,0)=\sin x+\cos x, x>0$$

34. Show that the function $u(x,y)=\sin(x-y)$ is a solution of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}.$$

Find the Fourier series of the following functions

35. $f(x)=\begin{cases}0 & \text{for } -3\le x\le 0\\ x & \text{for } 0\le x\le 3.\end{cases}$

36. $f(x)=e^x$ for $0\le x\le 2.$

37. $f(x)=x,\ -\pi<x<\pi.$

38. $f(x)=|\sin x|.$

39. $f(x)=\begin{cases}x & \text{for } 0\le x\le \frac{1}{2}\\ 1-x & \text{for } \frac{1}{2}<x\le 1.\end{cases}$

40. $f(x)=x-x^2$ from $x=-\pi$ to $x=\pi.$

41. $f(x)=x^2, 0\le x\le \pi.$

42. Discuss the solution of the Black –Scholes model of financial Mathematics/ Financial Engineering.

43. Solve Exercise 16 of Chapter 12(Webct).

44. Solve Exercise 17 of Chapter 12(Webct).

45. Solve Exercise 18 of Chapter 12(Webct).

46. Solve Exercise 19 of Chapter 12(Webct).

47. Solve Exercise 20 of Chapter 12(Webct).

48. Solve Problems 1-3 on page 291 of the prescribed book

49. Solve Problems 3 on page 223 of the prescribed book.

50. Solve Problems 5 and 6 on page 187 of the prescribed book.

51. Solve Problems 3 on page 187 of the prescribed book.

52. Solve the following exercises of Chapter 12(Webct).

53. Exercise 22.

54. Exercise 23.

55. Exercise 24.

56. Exercise 25.

57. Exercise 28.

58. Exercise 31.