

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 551 (ABSTRACT ALGEBRA)

SEMESTER 072 (SPRING 2008)

DR. JAWAD ABUHLAIL

1) **Description:** *Basic Definitions of Rings and Modules, Homomorphisms, Sums and Products, Exactness, Hom and Tensor, Adjoint Isomorphisms, Free, Projective and Injective Modules. Chain Conditions, Primary Decomposition, Noetherian Rings and Modules, Artinian Rings, Structure theorems.*

2) **Prerequisite:** Math 345 (Math 450 is recommended)

3) **TEXTBOOKS:**

- P. Grillet, *Abstract Algebra*, 2nd edition, Springer (2007).
- S. Lang, *Algebra*, Revised 3rd edition, Springer (2005).

4) **FURTHER READING:**

- N. Bourbaki, *Elements of mathematics: Algebra I*, Springer (1988).
- D.S. Dummit and R.M. Foote, *Abstract Algebra*, 3rd edition, John Wiley & Sons (2004).
- T. Hungerford, *Algebra*, Graduate Texts in Mathematics 73, Springer-Verlag, New York-Berlin (1980).

5) **GRADING POLICY:**

Exam I	200
Exam II	200
Research Project	100
Total	500

Detailed Syllabus¹

Week(s)	Section(s)	Title	Highlights
Chapter III. Rings			
1-2	III.1-III.2	Rings, Subrings and Ideals	Definitions & examples
	III.3	Homomorphisms	Theorem 3.6.
	III.4	Domains and Fields	4.5. – 4.7., 4.11.
3- 4	III.5 & III.6	Polynomials in One & Several Variables	5.1., 5.5., 5.7., 5.11., 5.12.; 6.8.
	III.7* ²	Formal Power Series	7.3., 7.5.
	III.8	Principal Ideal Domains	8.3., 8.4., 8.11., 8.12.
5	III.10	Unique Factorization Domains	10.2., 10.4., 10.9.
	III.11	Noetherian Rings	11.1., 11.2. (Hilbert's Basis Theorem)
Chapter VII. Commutative Rings			
6	VII.	Primary Decomposition	1.9., 1.10. (Noether-Lasker)
	VII.7	Localization	4.2., 4.5., 4.7., 4.10.
Chapter VIII. Modules			
7	VIII.1.	Definition	Definitions and examples
	VIII.2.	Homomorphisms	Theorems 2.6. – 2.8.
8	III.3.	Direct Sums and Products	3.1., 3.2., 3.5.
	III.4 & III.5	Free Modules / Vector Spaces	4.2., 4.5., 4.6. / 5.3.; 5.5.
9	III.6	Modules over Principal Ideal Domains	6.1. – 6.3.
10	III.8	Chain Conditions (Modules of Finite Length)	Equivalent conditions defining Noetherian/Artinian Modules 8.8., 8.10
Chapter IX. Semisimple Rings and Modules			
11	IX.1	Simple Rings and Modules	1.2.(Schur's Lemma), 1.8., 1.9.
	IX.2	Semisimple Modules	2.1.
	IX.3	The Artin Wedderburn Theorem	3.1., 3.3., 3.8.
12	IX.4*	Primitive Rings	Jacobson Density Theorem
	IX.5	The Jacobson Radical	5.1., 5.2., 5.6., 5.6. & 5.7. (Nakayama's Lemma)
	IX.6	Artinian Rings	6.1. – 6.4. (Hopkins-Levitzki)
Chapter X. Projectives and Injectives			
13	X.1	Exact Sequences	Five-Lemma, Nine-Lemma
	X.2*	Pullbacks and Pushouts	Existence/Uniqueness & Basic Properties
14	X.3	Projective Modules	3.2., 3.6., 3.7.
	X.4	Injective Modules	4.1., 4.5. (Baer's Criterion), 4.7., 4.11., 4.12.
	X.5*	The Injective Hull	5.6. (Existence & Uniqueness)
	X.6*	Hereditary Rings	Definition, Dedekind Domains
Chapter XI. Construction			
15-16	XI.1	Groups of Homomorphisms	1.4., 1.5.
	XI.2	Properties of Hom	2.1., 2.5.
	XI.5 & XI.6	Tensor Products and their Properties	Existence/Uniqueness; 6.6., 6.7., 6.9.
	XI.7*	Dual Modules	7.4., 7.5., 7.7.
	XI.8	Flat Modules	Basic Properties, 8.7. 8.11.

Numbers refer to Grillet's book "Abstract Algebra"¹
Sections marked with "*" will be covered as projects by students²