1) **Description:** Basic Definitions of Rings and Modules, Homomorphisms, Sums and Products, Exactness, Hom and Tensor, Adjoint Isomorphisms, Free, Projective and Injective Modules. Chain Conditions, Primary Decomposition, Noetherian Rings and Modules, Artinian Rings, Structure theorems.

2) **Prerequisite:** Math 345  (Math 450 is recommended)

3) **Textbooks:**

4) **Further Reading:**

5) **Grading Policy:**

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Detailed Syllabus

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<td>III.7*</td>
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<td>III.8</td>
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<td>III.11</td>
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<td>11.1., 11.2. (\text{(Hilbert’s Basis Theorem)})</td>
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**Chapter VII. Commutative Rings**

| 6       | VII.       | Primary Decomposition | 1.9., 1.10. (Noether-Lasker) |
|         | VII.7      | Localization | 4.2., 4.5., 4.7., 4.10. |

**Chapter VIII. Modules**

| 7       | VIII.1.    | Definition | Definitions and examples |
| 8       | III.3.     | Direct Sums and Products | 3.1., 3.2., 3.5. |
|         | III.4 & III.5| Free Modules / Vector Spaces | 4.2., 4.5., 4.6., 5.3., 5.5. |
| 9       | III.6.     | Modules over Principal Ideal Domains | 6.1. – 6.3. |
| 10      | III.8.     | Chain Conditions | Equivalent conditions defining Noetherian/Artinian Modules | 8.8., 8.10 |

**Chapter IX. Semisimple Rings and Modules**

| 11      | IX.1.      | Simple Rings and Modules | 1.2.(Schur’s Lemma), 1.8., 1.9. |
|         | IX.2.      | Semisimple Modules | 2.1. |
|         | IX.3.      | The Artin Wedderburn Theorem | 3.1., 3.3., 3.8. |
| 12      | IX.4*      | Primitive Rings | Jacobson Density Theorem |
|         | IX.5.      | The Jacobson Radical | 5.1., 5.2., 5.6., 5.6. & 5.7. \(\text{(Nakayama’s Lemma)}\) |
|         | IX.6.      | Artinian Rings | 6.1. – 6.4. (Hopkins-Levitzki) |

**Chapter X. Projectives and Injectives**

| 13      | X.1.       | Exact Sequences | Five-Lemma, Nine-Lemma |
|         | X.2*       | Pullbacks and Pushouts | Existence/Uniqueness & Basic Properties |
|         | X.5*       | The Injective Hull | 5.6. (Existence & Uniqueness) |
|         | X.6*       | Hereditary Rings | Definition, Dedekind Domains |

**Chapter XI. Construction**

| 15-16   | XI.1.      | Groups of Homomorphisms | 1.4., 1.5. |
|         | XI.2.      | Properties of Hom | 2.1., 2.5. |
|         | XI.7*      | Dual Modules | 7.4., 7.5., 7.7. |
|         | XI.8       | Flat Modules | Basic Properties, 8.7. 8.11. |

Numbers refer to Grillet’s book “Abstract Algebra”\(^1\)
Sections marked with “*” will be covered as projects by students \(^2\)