

## **Math 654 (in Semester 112)**

### **“Advanced Topics in Algebra”**

**by Jawad Abuhlail**

**Rationale/Objectives:** The course provides the students with a geometric motivation and interpretations of a number of the basic results in commutative Algebra. On the other hand, it provides the students with the basic tools needed to begin studying Algebraic Geometry.

**Text Book:** David Eisenbud, *Commutative Algebra: with a View toward Algebraic Geometry* (Graduate Texts in Mathematics), Springer (2004).

#### **Further Reading:**

- 1) M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Addison-Wesley Publishing Co. (1969).
- 2) N. Bourbaki, *Commutative Algebra*, Chapters 1-7, Springer (1998).
- 3) L. Rowen, *Graduate Algebra: Commutative View*, Graduate Studies in Mathematics 73, AMS (2006).

#### **Grading Policy:**

<b>Assignments</b>	<b>Midterm</b>	<b>Final</b>
30%	30%	40%

## Syllabus

Chapter	Section	Details	Weeks
<b>Ch. 0</b>		<b>Elementary Definitions</b>	<b>1</b>
	0.1	Rings and Ideals	
	0.2	Unique Factorization	
	0.3	Modules	
<b>Ch. 1</b>		<b>Roots of Commutative Algebra</b>	<b>5</b>
	1.1	Number Theory	
	1.2	Algebraic Curves and Function Theory	
	1.3	Invariant Theory	
	1.4	The Basis Theorem	
	1.6	Algebra and Geometry: The Nullstellensatz	
	1.7	Geometric Invariant Theory	
	1.8	Projective Varieties	
	1.9	Hilbert Functions and Polynomials	
	1.10	Free Resolutions and the Syzygy Theorem	
<b>Ass. 1</b>	Exercises	Noetherian Rings and Modules	
<b>Ass. 2</b>	Exercises	Algebra and Geometry	
<b>Ass. 3</b>	Exercises	Free Resolutions	
<b>Ass. 4</b>	Exercises	Spec, max-Spec and the Zariski Topology	
<b>MID TERM EXAM (30 %)</b>			
<b>Ch. 2</b>		<b>Localization</b>	<b>3</b>
	2.1	Fractions	
	2.2	Hom and Tensor	
	2.3	The Construction of Primes	
	2.5	Products of Domains	
<b>Ass. 5</b>	Exercises	Constructing Primes	
<b>Ass. 6</b>	Exercises	Idempotents, Products and Connected Components	
<b>Ch. 3</b>		<b>Associated Primes and Primary Decomposition</b>	<b>6</b>
	3.1	Associated Primes	
	3.3	Primary Decomposition	
	3.4	Primary Decomposition and Factoriality	
	3.6	Extracting Information from Primary Decomposition	
	3.7	Why is Primary Decomposition not Unique?	
	3.8	Geometric Interpretation of Primary Decomposition	
	3.9	Symbolic Powers and Functions Vanishing to Higher Order	
<b>Ass. 7</b>	Exercises	Total Quotients	
<b>Ass. 8</b>	Exercises	Prime Avoidance	
<b>FINAL EXAM (40 %)</b>			