

**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**  
**DEPARTMENT OF MATHEMATICS & STATISTICS**  
**DHAHRAN, SAUDI ARABIA**

**STAT 416 : Stochastic Processes for Actuaries (122)**

**Instructor:** Adnan Jabbar

**Office:** Building 5-room 320

**Phone:** 7630

**E-mail:** [ajabbar@kfupm.edu.sa](mailto:ajabbar@kfupm.edu.sa)

**Office Hours:** Saturday: 8:00 am - 9:00 am, 12:05 pm - 1:05 pm.

Monday: 8.00am – 9.00am, 12.05pm - 1.05pm

Wednesday: 8.00am – 9.00am, 12.05pm-1.05pm or by appointment

**Textbook:**

1. Introduction to Probability Models, 10<sup>th</sup> edition, by Sheldon M. Ross (2012)

**Course Objectives:**

Basic classes of stochastic processes. Poisson (regular, compound, compound surplus, and non-homogenous) and renewal processes with applications in simple queuing systems and Actuarial Science. Discrete and continuous time Markov chains. Birth-Death and Yule processes. Branching models of population growth processes. Actuarial risk models; simulation. Arithmetic and geometric Brownian motions, and applications of these processes such as in computation of resident fees for continuing care retirement communities, and pricing of financial instruments.

**Assessment**

Assessment for this course will be based on attendance, homework, two major exams and a comprehensive final exam, as in the following:

| Activity  | Weight          |
|---|-----------------|
| Quiz , Homework and attendance  | (12% + 6% + 2%) |
| Exam 1 (Chapters 1, 2, 3, 4)<br><b>Tuesday February 26, 2013 ,7pm, in Building 59</b> | 20%             |
| Exam 2 (Chapters 5, 6 & 7)<br><b>Tuesday April 23, 2013, 7pm, in Building 59</b>      | 20%             |
| Final Exam (Comprehensive)  | 40%             |

**\*You need to achieve at least 50% in order to pass the course**

**Academic Integrity:** All KFUPM policies regarding **ethics** and **academic honesty** apply to this course.

**Important Notes:**

- ✓ Excessive unexcused absences will result in a grade of **DN** in accordance with University rules.
- ✓ **Attendance** on time is **very** important.
- ✓ **A formula sheet** and **statistical tables** will be provided for you in every exam.

**Home Work:**

- Handout problems will be posted on the WebCT or in the instructor home page towards the end of each chapter.
- The **Homework** should be submitted in the first Saturday after completing the chapter **and no need for an announcement in advance**.
- No late homework will be accepted.

### Syllabus (Tentative)

| <i>Week</i>  | <i>Sections</i> | <i>Topics</i>  | <i>Notes</i> |
|--|-----------------|--|--------------|
| <b>Week 1</b><br>26/1 – 30/1   | 1.1-1.6         | Basic concepts from probability theory (Review)                | 29 January   |
| <b>Week 2</b><br>2/2 – 6/2   | 2.1-2.9         | Random Variables and Conditional probability                   |              |
| <b>Week 3</b><br>9/2 – 13/2  | 3.1-3.7         | Stochastic processes and introduction to the Markov chains     |              |
| <b>Week 4</b><br>16/2 – 20/2   | 4.1-4.5         | Markov chains  |              |
| <b>Week 5</b><br>23/2 – 27/2   | 4.6-4.11        | Markov chains – Branching Processes                            |              |
| <b>Week 6</b><br>2/3 – 6/3   | 5.1-5.3         | The Exponential Distribution and The Poisson Process           |              |
| <b><u>February 26 - 1-st Major Exam</u> (chapters 1, 2, 3 &amp; 4)</b> |                 |  |              |
| <b>Week 7</b><br>9/3 – 13/3  | 5.4-6.4         | The Poisson Process and Continuous-Time Markov Chains          |              |
| <b>Week 8</b><br>16/3 – 20/3   | 6.5-6.8         | Continuous-Time Markov Chains                                  |              |
| <b><u>Midterm Vacation</u> March 23-27, 2013</b>                       |                 |  |              |
| <b>Week 9</b><br>30/3 – <b>3/4</b>                                     | 7.1-7.5         | Birth and Death Processes and Transition Probability Function  |              |
| <b>Week 10</b><br>6/4 – 10/4   | 7.6-7.10        | Queuing Theory-Exponential Models                              |              |
| <b>Week 11</b><br>13/4 – 17/4  | 8.1-8.6         | Brownian Motion-Geometric Brownian Motion                      |              |
| <b>Week 12</b><br>20/4 – 24/4  | 8.7-9.3         | Stochastic Models for stock prices and interest rates          |              |
| <b><u>April 23 - 2-nd Major Exam</u> (chapters 5, 6 &amp; 7)</b>       |                 |  |              |
| <b>Week 13</b><br>27/4 – <b>1/5</b>                                    | 9.4-10.3        | Pricing Stock Options-The Arbitrage Theorem-Gaussian Processes |              |
| <b>Week 14</b><br>4/5 – 8/5  | 10.4-11.3       | General Techniques for Simulating Continuous Random Variables  |              |
| <b>15</b><br>11/5 – 15/5   | 11.4-11.8       | Stochastic Processes- Simulating from Discrete Distribution    |              |
| <b><u>Comprehensive Final Exam</u> , May 16, 2013</b>                  |                 |  |              |

## **Learning objectives and outcomes:**

### **On completion of the course the students will be able to:**

Explain the concepts of probability.

- . Explain what is meant by a set function, a sample space for an experiment, and an event.
- Define probability as a set function on a collection of events, stating basic axioms.
- . Derive basic properties satisfied by the probability of occurrence of an event, and calculate probabilities of events in simple situations.
- . Derive the addition rule for the probability of the union of two events, and use the rule to calculate probabilities.
- . Define the conditional probability of one event given the occurrence of another event, and calculate such probabilities.
- . Derive Bayes' Theorem for events, and use the result to calculate probabilities.

Describe the properties of Poisson processes:

- For increments in the homogeneous case
- For interval times in the homogeneous case
- Resulting from special types of events in the Poisson process
- Resulting from sums of independent Poisson processes.
- For any Poisson process and the interarrival and waiting distributions associated with the Poisson process.
- Calculate: Expected values  $\bar{x}$  Variances  $\bar{v}$  Probabilities. For a compound Poisson process, calculate moments associated with the value of the process at a given time.
- Discuss the properties of the nonhomogeneous Poisson process.

Describe the properties of the exponential distribution:

- Discuss the properties of the exponential distribution.

Describe the properties of discrete and continuous Markov chains:

- For homogenous discrete-time Markov chain models: Define each model. Calculate probabilities of being in a particular state at a particular time.
- Calculate probabilities of transitioning between states.
- Explain what is meant by the Markov property in the context of a stochastic process.
- Define and apply a Markov chain.
- State the essential features of a Markov chain model.
- Applications of the Gambler's ruin problem.
- Markov chains Branching process.
- Hidden Markov Chains.
- State the features of the Continuous-time Markov chains.
- Discuss the features of the Birth and death processes.

- Demonstrate how Markov chains can be used as a tool for modelling and how they can be simulated.
- Describe a system of frequency based experience rating in terms of a Markov chain and describe other simple applications.
- Describe a time-inhomogeneous Markov chain model and describe simple applications.

Demonstrate the knowledge and understanding of Queuing theory.

- Discuss the steady state probabilities
- Discuss the applications of the Birth and Death Queuing Models.

Demonstrate a knowledge and understanding of stochastic models of the behaviour of stock prices.

- Discuss the continuous time log-normal model of security prices and the empirical evidence for or against the model.

Define and apply the main concepts of stochastic process Brownian motion (or Wiener Processes).

- Explain the definition and basic properties of standard Brownian motion or Wiener process.
- Demonstrate a basic understanding of stochastic differential equations, the Ito integral, diffusion and mean-reverting processes.
- State Ito's formula and be able to apply it to simple problems.
- Write down the stochastic differential equation for geometric Brownian motion and show how to find its solution.
- Show an understanding of simple stochastic models for investment returns.
- Discuss the applications of pricing stock options.
- State the Arbitrage theorem.
- Discuss the applications of geometric Brownian motion.

Describe the general techniques for simulating continuous random variables.

- Describe the generation of random variates from specified distribution.
- Discuss the inverse transformation method.
- Discuss the applications of acceptance and rejection methods.
- Disadvantages of using truly random numbers.
- Common sets of random numbers versus independent sets of random numbers.