

**SYLLABUS**

Semester II: 2013-2014(132)

**Instructor:** Dr. A. Bonfoh  
**Course #:** MATH 590 Special Topics in Mathematics  
**Title:** Infinite-dimensional Dynamical Systems

**Objectives:** The course aims to solve and describe the large time behavior of solutions to nonlinear partial differential equations in infinite-dimensional phase-spaces.

**Course Description:** This course will introduce recent research topics in infinite-dimensional dynamical systems associated with partial differential equations. The global attractor theory for dissipative evolution equations will be discussed in detail. The course will cover Sobolev and Hilbert spaces, existence and uniqueness of solutions, Semigroups, limits sets, the global attractor, continuity of the global attractor, finite dimensionality of attractors, exponential attractors, inertial manifolds. As typical examples, attractors for reaction-diffusion equations and Navier-Stokes equations will be studied.

**Textbook:** J.C. Robinson, *Infinite-dimensional Dynamical systems*, Cambridge University Press, Cambridge, 2001

- References:**
1. R. Temam, *Infinite-dimensional dynamical systems in Mechanics and Physics*, Second Edition, Springer-Verlag, New York, 1997
  2. J. Hale, *Asymptotic behavior of Dissipative Systems*, Providence R.I., 1988.
  3. A.V. Babin, M.I. Vishik, *Attractors of Evolution Equations*, North Holland, Amsterdam, 1991.
  4. A. Eden, C. Foias, B.Nicolaenko and R. Temam, *Exponential attractors for dissipative evolution equations*, Masson, Paris, 1994.

Week	Date	Sec.	Topics	Suggested Homework Problems
1	Jan. 26-30	5.2 5.7 6.1 6.2 6.3	General Sobolev spaces The Sobolev embedding theorem Classical, strong and weak solutions Weak solution of Poisson's equation Higher regularity for the Laplacian	
2	Feb. 2-6	7.1 8.1 8.2 8.3 8.4	Banach spaces valued function spaces Nonlinear Reaction-Diffusion Equation The Basis for the Galerkin Expansion Weak solutions Strong solutions	
3	Feb. 9-13	9.1 9.2 9.3 9.4 9.5 9.6	The Stokes operator The weak form of the Navier Stokes equation Properties of the Trilinear form Existence of weak solutions Unique solution in 2d Existence of strong solutions in 2d	
4	Feb. 16-20	10.1 10.2 10.3 10.4 10.5	Semigroups Dissipation Limits sets and attractors A theorem for the existence of global attractors An example- The Lorenz attractor	

		10.6	Structure of the attractor	
5	Feb. 23-27	11.1	Reaction-Diffusion Equation- Absorbing sets and the attractor	
		11.2	Regularity results	
		11.4	A Lyapunov functional	
		11.5	The Chaffee-Infante equation	
<b>First Exam</b>				
6	Mar. 2-6	12.1	Attractors for 2d Navier-Stokes equation	
7	Mar. 9-13	12.2	Attractors for The 3d Navier-Stokes equation	
8	Mar. 16-20	10.8.1	Upper semicontinuity of the global attractor	
		10.8.2	Lower semicontinuity of the global attractor	
<b>Midterm Vacation</b>				
9	Mar. 30-Apr. 3	13.1	Fractal and Hausdorff dimensions	
		13.2	Bounding the attractor dimension	
10	Apr. 6-10	13.3	Example 1: The Reaction-Diffusion Equation	
		13.4	Example 2: The 2d Navier-Stokes Equation	
<b>Second Exam</b>				
11	Apr. 13-17	14.1	The squeezing property	
		14.4	The squeezing property for Reaction-Diffusion Equations	
12	Apr. 20-24	14.5	The 2d Navier Stokes equations	
		14.6	Finite-dimensional Exponential attractors	
13	Apr. 27-May 1	15.2.1	The strong squeezing property	
		15.2.2	Inertial manifolds	
14	May. 4-8	15.4	Inertial manifolds for Reaction-Diffusion Equation	
15	May 11-15	15.5.1	Inertial manifolds and the 2d Navier-Stokes equation	
<b>Final Exam</b>				

**Grading:** The distribution of grade is as follows:

Class test I, II	30%
Class work (homework assignments)	30%
Final Exam	40%
 Total	 100%