KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT416 : Stochastic Processes for Actuaries (132)

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Textbook:

1. Introduction to Probability Models, 10th edition, by Sheldon M. Ross (2012)

Course Objectives:

Basic classes of stochastic processes. Poisson (regular, compound, compound surplus, and non-homogenous) and renewal processes with applications in simple queuing systems and Actuarial Science. Discrete and continuous time Markov chains. Birth-Death and Yule processes. Branching models of population growth processes. Actuarial risk models; simulation.Arithmetic and geometric Brownian motions, and applications of these processes such as in computation of resident fees for continuing care retirement communities, and pricing of financial instruments.

Assessment

Assessment for this course will be based on attendance, homework, two major exams and a comprehensive final exam, as in the following:

Activity	Weight
Quiz , Homework	14%
Exam 1 (Chapters 4 & 5) **March 8, 2015-Class time**	16%
Exam 2 (Chapters 6 & 7) **April 9, 2015- Class time**	16%
Exam 2 (Chapters 8 & 9) **May 3, 2015-Class time**	16%
Final Exam (Comprehensive) **Tuesday, May 26 at 8:00 am (as posted on registrar website)**	38%

***You need to achieve at least 50% in order to pass the course**

Academic Integrity: All KFUPM policies regarding **ethics** and **academic honesty** apply to this course.

Important Notes:

- ✓ Excessive unexcused absences will result in a grade of ***DN*** in accordance with University rules.
- ✓ ***Attendance*** on time is ***very*** important.
- ✓ *A formula sheet* and *statistical tables* will be provided for you in every exam.

Home Work:

- ➢ Handout problems will be posted on the WebCT or in the instructor home page towards the end of each chapter.
- ➢ The ***Homework*** should be submitted in the first Saturday after completing the chapter ***and no need for an announcement in advance***.
- ➢ No late homework will be accepted.

Syllabus **(Tentative)**

Week	*Sections*	**Topics**	*Notes*
Week 1 Jan 25 –29	1.1-1.6 2.1 – 2.9 3.1 – 3.5	Introduction to Probability theory (review) Random variables (review) Conditional Probability and Conditional Expectation (review)	
Week 2 Feb 1 – 5	4.1 – 4.3	Introduction, Chapman – Kolmogorov Equation Classification of States	
Week 3 Feb 8 – 12	4.4-4.8	Limiting Probabilities, Some applications, Mean time Spent in Transient States, Branching Processes, Time Reversible Markov Chains,	
Week 4 Feb 15 – 19	5.1 – 5.3	Introduction, The Exponential distribution and The Poisson Processes	
Week 5 Feb 22 – 26	5.3-5.4	The Poisson Processes (cont) and Generalization of the Poisson Process	
Week 6 Mar 1 – 5	6.1 – 6.4	Introduction Continuous-Time Markov Chains, Birth and Death Processes and The Transition Probability Function $P_{ij}(t)$	
Week 7 Mar 8 – 12	6.5-6.8	Limiting Probabilities, Time Reversibility, Uniformization and Computing the Transition Probabilities	
Week 8 Mar 15 – 19	7.1-7.5	Introduction, Distribution of $N(t)$, Limit Theorems and their Applications, renewal Reward Processes and Regenerative Processes	
Midterm Vacation **March 22-26, 2015**			
Week 9 Mar 29 – Apr 2	7.6-7.10	Semi – Markov Processes, the Inspection Paradox, Computing the Renewal Function, Applications to Patterns and The Insurance Ruin Problem	
Week 10 Apr 5 – 9	8.1-8.7	Introduction, Preliminaries, Exponential Models, Network of Queues, The System M/G/1, Variation on the M/G/1 and the Model G/M/1	
Week 11 Apr 12 – 16	9.1 – 9.3	Introduction, Structure Functions, Reliability of Systems of Independent Components and Bounds of the Reliability Function	
Week 12 Apr 19 – 23	9.5-9.7	System Life as Function of Component Lives, Expected System Lifetime and System with Repair	
Week 13 Apr 26 – 30	10.1- 10.4 &5-7	Brownian Motion, Hitting Times, Variations on Brownian Motion and Pricing stock Options	
Week 14 May 3 – 7	4.9, 11.1- 11.8	Markov-Chain Monte Carlo, Simulation (emphasis on 11.5 onwards)	
Week 15 May 10 – 14	Review	Simulation (cont.) Review	
Comprehensive Final Exam			

Learning objectives and outcomes:

On completion of the course the students will be able to:

Explain the concepts of probability.

- Explain what is meant by a set function, a sample space for an experiment, and an event.
- Define probability as a set function on a collection of events, stating basic axioms.
- Derive basic properties satisfied by the probability of occurrence of an event, and calculate probabilities of events in simple situations.
- Derive the addition rule for the probability of the union of two events, and use the rule to calculate probabilities.
- Define the conditional probability of one event given the occurrence of another event, and calculate such probabilities.
- Derive Bayes' Theorem for events, and use the result to calculate probabilities.

Understand the concepts of random variables.

- Define stochastic process.
- Use limit theorems

Understand the concepts of conditioning and compounding.

- Define compound random variables.
- Compute expectations by conditioning
- Use variance formula by conditioning

Describe the properties of Poisson processes:

- For increments in the homogeneous case
- For interval times in the homogeneous case
- Resulting from special types of events in the Poisson process
- Resulting from sums of independent Poisson processes.
- For any Poisson process and the interarrival and waiting distributions associated with the Poisson process.
- Calculate: a) Expected values b) Variances c) Probabilities. For a compound Poisson process, calculate moments associated with the value of the process at a given time.
- Discuss the properties of the **nonhomogeneous Poisson** process.
- Discuss the properties of the **Compound Poisson** process.
- Describe the properties of the exponential distribution

Describe the properties of discrete and continuous Markov chains:

- For homogenous discrete-time Markov chain models: Define each model. Calculate probabilities of being in a particular state at a particular time.
- Calculate probabilities of transitioning between states.
- Explain what is meant by the Markov property in the context of a stochastic process.
- Define and apply a Markov chain.
- State the essential features of a Markov chain model.
- Applications of the Gambler's ruin problem.
- State the features of the Continuous-time Markov chains.
- Discuss the features of the Birth and death processes.
- Demonstrate how Markov chains can be used as a tool for modelling and how they can be simulated.
- Describe a system of frequency based experience rating in terms of a Markov chain and describe other simple applications.
- Describe a time-inhomogeneous Markov chain model and describe simple applications.

Describe the properties of a Branching Process.

Demonstrate the knowledge and understanding of Queuing theory.

- Discuss the steady state probabilities
- Discuss the applications of the Birth and Death Queuing Models.

Demonstrate understanding of stochastic models of the behavior of stock prices.

- Discuss the continuous time log-normal model of security prices and the empirical evidence for or against the model.

Define and apply the main concepts of stochastic process Brownian motion (or Wiener Processes).

- Explain the definition and basic properties of standard Brownian motion or Wiener process.
- Write down the stochastic differential equation for geometric Brownian motion and show how to find its solution.
- Show an understanding of simple stochastic models for investment returns.
- Discuss the applications of pricing stock options.
- State the Arbitrage theorem.
- Discuss the applications of **geometric Brownian** motion.

Describe the general techniques for simulating continuous random variables.

- Describe the generation of random variates from specified distribution.
- Discuss the inverse transformation method.
- Discuss the applications of acceptance and rejection methods.
- Disadvantages of using truly random numbers.
- Common sets of random numbers versus independent sets of random numbers.