Math 668

Evolution Equations

Introduction: Partial Differential Equations arise in many fields of science and engineering such as physics, mechanics, material science, image processing, finance math, … etc.
Over years, mathematicians and scientists, in general, made great efforts to find ways and methods to solve these equations. One powerful tool is the Functional Analysis, which is used to choose the right set of solutions (Admissible space) and the appropriate method to obtain the solution. Sobolev spaces prove to be the most efficient class of functions to tackle such problems.
In **Math 569** (Linear Elliptic PDE's), appropriate functional Analysis tools are introduced as well as application to stationary problems are given
In this course, it is intended to introduce students to more Functional Analysis approach to solving PDE's. More precisely PDE's of evolutionary type. This course is a natural continuation of the course dealing with stationary problems (elliptic equations).

Course Description Maximum Monotone Operators, Bounded and unbounded operators, Pseudo monotone operators, Self-adjoint, Evolution Equations in Hilbert and Banach spaces, Hille-Yosida Theorem, application to linear heat and wave Equations, Nonlinear Evolution equations, The Galerkin Method

Textbook: *Functional Analysis, Sobolev spaces, and PDE's, Haim Brezis, Springer, Universitex 2011*

References Partial Differential Equations, L.C. Evans (AMS), Second Edition.

Objectives: This course is intended to introduce students to the functional analysis approach and modern theory of solving evolution equations and prepare them for further studies in the subject.

Outcomes: By the end of the course, the students should be able to use the modern ideas of Functional Analysis to

1) Deal comfortably with monotone operators
2) Solve some linear and nonlinear evolution problems
3) Have a vision to future research

Syllabus

Week #	Chapter	Section	HW
1	**Chapter 5: Hilbert Spaces: Review**	5.1 Definitions and elementary Properties 5.2. Dual of a Hilbert space	
2		5.3 Theorems of Stampacchia and Lax-Milgram 5.4. Orthogonal bases	5.6, 5.7, 5.8, 5.11, 5.26
3	**Chapter 7: Hille-Yosida Theorem**	7.1 Maximal Monotone Operators in Hilbert spaces	
4-5		7.2. Existence and Uniqueness	
6		7.3 Regularity.	
7		7.4. Self-Adjoint case	
8-9	**Chapter 10: The Heat Equation and the Wave equation**	10.1. **The Heat Equation:** Existence, Uniqueness and regularity	
10		10.2. The maximum Principle	
11-12		10.3. **The Wave equation:** Existence, Uniqueness and regularity	
13	**Additional material**	Maximal Monotone Operators in Banach spaces	
14		Nonlinear Nonlinear Evolution Equations: Galerkin method	
15		Application to Nonlinear Heat and Wave equations	