

**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**  
**DEPARTMENT OF MATHEMATICS & STATISTICS**  
**DHAHRAN, SAUDI ARABIA**

**STAT416: Stochastic Processes for Actuaries (161)**

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**Office Hours:** UTR: 10.00 -10.50 or by appointment

**Textbook:**

Introduction to Probability Models, 10<sup>th</sup> edition, by Sheldon M. Ross (2012)

**Course Objectives:**

Basic classes of stochastic processes. Poisson (regular, compound, compound surplus, and non-homogenous) and renewal processes with applications in simple queuing systems and Actuarial Science. Discrete and continuous time Markov chains. Birth-Death and Yule processes. Branching models of population growth processes. Actuarial risk models; simulation. Arithmetic and geometric Brownian motions, and applications of these processes such as in computation of resident fees for continuing care retirement communities, and pricing of financial instruments.

**Assessment**

Assessment for this course will be based on attendance, homework, two major exams and a comprehensive final exam, as in the following:

Activity	Weight
Homework	15%
Exam 1 (Chapters 4 & 5) <b>October 27, 2016-Class time</b>	18%
Exam 2 (Chapters 6 & 7) <b>December 4, 2016- Class time</b>	16%
Exam 2 (Chapters 8 & 9) <b>January 1, 2017-Class time</b>	16%
Final Exam (Comprehensive) <b>TBA</b>	35%

**\*You need to achieve at least 50% in order to pass the course**

**Academic Integrity:** All KFUPM policies regarding **ethics** and **academic honesty** apply to this course.

**Important Notes:**

- ✓ Excessive unexcused absences will result in a grade of **DN** in accordance with University rules.
- ✓ **Attendance** on time is **very** important.
- ✓ **A formula sheet** and **statistical tables** will be provided for you in every exam.

**Home Work:**

- Handout problems will be posted on the WebCT or in the instructor home page towards the end of each chapter.
- The **Homework** should be submitted in the first Saturday after completing the chapter ***and no need for an announcement in advance.***
- No late homework will be accepted.

### Syllabus (Tentative)

Week	Sections	Topics
<b>1</b> Sept.18-21	1.1-1.6 2.1 – 2.9 3.1 – 3.5	Introduction to Probability theory ( review) Random variables (review) Conditional Probability and Conditional Expectation (review)
<b>2</b> Sept.25-29	4.1 – 4.3	Introduction, Chapman – Kolmogorov Equation Classification of States
<b>3</b> Oct. 2- 6	4.4-4.8	Limiting Probabilities, Some applications, Mean time Spent in Transient States, Branching Processes, Time Reversible Markov Chains,
<b>4</b> Oct. 9- 13	5.1 – 5.3	Introduction, The Exponential distribution and The Poisson Processes
<b>5</b> Oct. 16-20	5.3-5.4	The Poisson Processes (cont.) and Generalization of the Poisson Process
<b>6</b> Oct. 23- 27	6.1 – 6.4	Introduction Continuous-Time Markov Chains, Birth and Death Processes and The Transition Probability Function $P_{ij}(t)$
<b>7</b> Oct. 30- Nov.3	6.5-6.8	Limiting Probabilities, Time Reversibility, Uniformization and Computing the Transition Probabilities
<b>8</b> Nov. 6- 10	7.1-7.5	Introduction, Distribution of $N(t)$ , Limit Theorems and their Applications, renewal Reward Processes and Regenerative Processes
<b>Nov. 13-17 Midterm Break</b>		
<b>9</b> Nov. 20- 24	7.6-7.10	Semi – Markov Processes, the Inspection Paradox, Computing the Renewal Function, Applications to Patterns and The Insurance Ruin Problem
<b>10</b> Nov.27- Dec.1	8.1-8.7	Introduction, Preliminaries, Exponential Models, Network of Queues, The System M/G/1, Variation on the M/G/1 and the Model G/M/1
<b>11</b> Dec. 4- 8	9.1 – 9.3	Introduction, Structure Functions, Reliability of Systems of Independent Components and Bounds of the Reliability Function
<b>12</b> Dec. 11- 15	9.5-9.7	System Life as Function of Component Lives, Expected System Lifetime and System with Repair
13 Dec. 18- 22	10.1- 10.4 &5-7	Brownian Motion, Hitting Times, Variations on Brownian Motion and Pricing stock Options
14 Dec. 25- 29	4.9, 11.1- 11.5	Markov-Chain Monte Carlo, Simulation (emphasis on 11.5 onwards)
15 Jan. 1- 5	11.6- 11.8	Variance Reduction Techniques, Determining number of runs, Generating from Stationary distribution
16 Jan.6		Review
<b>Comprehensive Final Exam</b>		

## Learning objectives and outcomes:

### **On completion of the course the students will be able to:**

Explain the concepts of probability.

- Explain what is meant by a set function, a sample space for an experiment, and an event.
- Define probability as a set function on a collection of events, stating basic axioms.
- Derive basic properties satisfied by the probability of occurrence of an event, and calculate probabilities of events in simple situations.
- Derive the addition rule for the probability of the union of two events, and use the rule to calculate probabilities.
- Define the conditional probability of one event given the occurrence of another event, and calculate such probabilities.
- Derive Bayes' Theorem for events, and use the result to calculate probabilities.

Understand the concepts of random variables.

- Define stochastic process.
- Use limit theorems

Understand the concepts of conditioning and compounding.

- Define compound random variables.
- Compute expectations by conditioning
- Use variance formula by conditioning

Describe the properties of Poisson processes:

- For increments in the homogeneous case
- For interval times in the homogeneous case
- Resulting from special types of events in the Poisson process
- Resulting from sums of independent Poisson processes.
- For any Poisson process and the interarrival and waiting distributions associated with the Poisson process.
- Calculate: a) Expected values b) Variances c) Probabilities. For a compound Poisson process, calculate moments associated with the value of the process at a given time.
- Discuss the properties of the **nonhomogeneous Poisson** process.
- Discuss the properties of the **Compound Poisson** process.
- Describe the properties of the exponential distribution

Describe the properties of discrete and continuous Markov chains:

- For homogenous discrete-time Markov chain models: Define each model. Calculate probabilities of being in a particular state at a particular time.
- Calculate probabilities of transitioning between states.
- Explain what is meant by the Markov property in the context of a stochastic process.
- Define and apply a Markov chain.
- State the essential features of a Markov chain model.

- Applications of the Gambler's ruin problem.
- State the features of the Continuous-time Markov chains.
- Discuss the features of the Birth and death processes.
- Demonstrate how Markov chains can be used as a tool for modelling and how they can be simulated.
- Describe a system of frequency based experience rating in terms of a Markov chain and describe other simple applications.
- Describe a time-inhomogeneous Markov chain model and describe simple applications.

Describe the properties of a Branching Process.

Demonstrate the knowledge and understanding of Queuing theory.

- Discuss the steady state probabilities
- Discuss the applications of the Birth and Death Queuing Models.

Demonstrate understanding of stochastic models of the behavior of stock prices.

- Discuss the continuous time log-normal model of security prices and the empirical evidence for or against the model.

Define and apply the main concepts of stochastic process Brownian motion (or Wiener Processes).

- Explain the definition and basic properties of standard Brownian motion or Wiener process.
- Write down the stochastic differential equation for geometric Brownian motion and show how to find its solution.
- Show an understanding of simple stochastic models for investment returns.
- Discuss the applications of pricing stock options.
- State the Arbitrage theorem.
- Discuss the applications of **geometric Brownian** motion.

Describe the general techniques for simulating continuous random variables.

- Describe the generation of random variates from specified distribution.
- Discuss the inverse transformation method.
- Discuss the applications of acceptance and rejection methods.
- Disadvantages of using truly random numbers.
- Common sets of random numbers versus independent sets of random numbers.