**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**
**DEPARTMENT OF MATHEMATICS & STATISTICS**
**DHAHRAN, SAUDI ARABIA**

# STAT416: Stochastic Processes for Actuaries (171)

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**Office Hours**: UTR: 1.00pm -1.50pm or by appointment

**Textbook:**

Introduction to Probability Models, 11th edition, by Sheldon M. Ross (2014)

**Course Objectives:**

Basic classes of stochastic processes. Poisson (regular, compound, compound surplus, and non-homogenous) and renewal processes with applications in simple queuing systems and Actuarial Science. Discrete and continuous time Markov chains. Birth-Death and Yule processes. Branching models of population growth processes. Actuarial risk models; simulation.Arithmetic and geometric Brownian motions, and applications of these processes such as in computation of resident fees for continuing care retirement communities, and pricing of financial instruments.

***Assessment***

Assessment for this course will be based on attendance, homework, two major exams and a comprehensive final exam, as in the following:

| Activity | Weight |
|---|---|
| Homework | 15% |
| Exam 1 (Chapters 4 & 5)<br>**October 26, 2017-Thursday** | 17% |
| Exam 2 (Chapters 6 & 7)<br>**November 23, 2017- Thursday** | 17% |
| Exam 3 (Chapters 8 & 9)<br>**December 14, 2017- Thursday** | 17% |
| Final Exam (Comprehensive)<br>**Sunday, December 31 , 2017 at 7:00 pm** | 34% |

***You need to achieve at least 50% in order to pass the course**

**Academic Integrity**: All KFUPM policies regarding **ethics** and **academic honesty** apply to this course.

***Important Notes:***

- ✓ Excessive unexcused absences will result in a grade of ***DN*** in accordance with University rules.
- ✓ ***Attendance*** on time is ***very*** important.
- ✓ *A formula sheet* and *statistical tables* will be provided for you in every exam.

**Home Work**:

- ➤ Handout problems will be posted on the WebCT or in the instructor home page towards the end of each chapter.
- ➤ The ***Homework*** should be submitted in the first Saturday after completing the chapter ***and no need for an announcement in advance***.
- ➤ No late homework will be accepted.

## **Syllabus (Tentative)**

| ***Week*** | ***Sections*** | **Topics** |
|---|---|---|
| **1**<br>Sep. 17- 21 | 1.1-1.6<br>2.1 – 2.9<br>3.1 – 3.5 | Introduction to Probability theory ( review)<br>Random variables (review)<br>Conditional Probability and Conditional Expectation (review) |
| **2**<br>Sep.24 -28 | 4.1 – 4.3 | Introduction, Chapman – Kolmogorov Equation<br>Classification of States |
| **3**<br>Oct. 1- 5 | 4.4-4.8 | Limiting Probabilities, Some applications, Mean time Spent in Transient States, Branching Processes, Time Reversible Markov Chains, |
| **4**<br>Oct. 7- 12 | 5.1 – 5.3 | Introduction, The Exponential distribution and The Poisson Processes |
| **5**<br>Oct. 15- 19 | 5.3-5.4 | The Poisson Processes (cont.) and Generalization of the Poisson Process |
| **6**<br>Oct. 22- 26 | 6.1 – 6.4 | Introduction Continuous-Time Markov Chains, Birth and Death Processes and The Transition Probability Function $P_{ij}(t)$ |
| **7**<br>Oct. 29- Nov.2 | 6.5-6.8 | Limiting Probabilities, Time Reversibility, Uniformization and Computing the Transition Probabilities |
| **8**<br>Nov. 5- 9 | 7.1-7.5 | Introduction, Distribution of $N(t)$, Limit Theorems and their Applications, renewal Reward Processes and Regenerative Processes |
| **9**<br>Nov 12- 16 | 7.6-7.10 | Semi – Markov Processes, the Inspection Paradox, Computing the Renewal Function, Applications to Patterns and The Insurance Ruin Problem |
| **10**<br>Nov.19- 23 | 8.1-8.7 | Introduction, Preliminaries, Exponential Models, Network of Queues, The System M/G/1, Variation on the M/G/1 and the Model G/M/1 |
| **11**<br>Nov. 26- 30 | 9.1 – 9.3 | Introduction, Structure Functions, Reliability of Systems of Independent Components and Bounds of the Reliability Function |
| **12**<br>Dec. 3- 7 | 9.5-9.7 | System Life as Function of Component Lives, Expected System Lifetime and System with Repair |
| 13<br>Dec. 10- 14 | 10.1-<br>10.4<br>&5-7 | Brownian Motion, Hitting Times, Variations on Brownian Motion and Pricing stock Options |
| 14<br>Dec. 17- 21 | 4.9,<br>11.1-<br>11.5 | Markov-Chain Monte Carlo,<br>Simulation (emphasis on 11.5 onwards) |
| 15<br>Dec. 24- 28 | 11.6-<br>11.8 | Variance Reduction Techniques, Determining number of runs, Generating from Stationary distribution, Review |
| Comprehensive Final Exam | | |

# Learning objectives and outcomes:

## On completion of the course the students will be able to:

Explain the concepts of probability.

- Explain what is meant by a set function, a sample space for an experiment, and an event.
- Define probability as a set function on a collection of events, stating basic axioms.
- Derive basic properties satisfied by the probability of occurrence of an event, and calculate probabilities of events in simple situations.
- Derive the addition rule for the probability of the union of two events, and use the rule to calculate probabilities.
- Define the conditional probability of one event given the occurrence of another event, and calculate such probabilities.
- Derive Bayes' Theorem for events, and use the result to calculate probabilities.

Understand the concepts of random variables.

- Define stochastic process.
- Use limit theorems

Understand the concepts of conditioning and compounding.

- Define compound random variables.
- Compute expectations by conditioning
- Use variance formula by conditioning

Describe the properties of Poisson processes:

- For increments in the homogeneous case
- For interval times in the homogeneous case
- Resulting from special types of events in the Poisson process
- Resulting from sums of independent Poisson processes.
- For any Poisson process and the interarrival and waiting distributions associated with the Poisson process.
- Calculate: a) Expected values b) Variances c) Probabilities. For a compound Poisson process, calculate moments associated with the value of the process at a given time.
- Discuss the properties of the **nonhomogeneous Poisson** process.
- Discuss the properties of the **Compound Poisson** process.
- Describe the properties of the exponential distribution

Describe the properties of discrete and continuous Markov chains:

- For homogenous discrete-time Markov chain models: Define each model. Calculate probabilities of being in a particular state at a particular time.
- Calculate probabilities of transitioning between states.
- Explain what is meant by the Markov property in the context of a stochastic process.
- Define and apply a Markov chain.
- State the essential features of a Markov chain model.
- Applications of the Gambler's ruin problem.

- State the features of the Continuous-time Markov chains.
- Discuss the features of the Birth and death processes.
- Demonstrate how Markov chains can be used as a tool for modelling and how they can be simulated.
- Describe a system of frequency based experience rating in terms of a Markov chain and describe other simple applications.
- Describe a time-inhomogeneous Markov chain model and describe simple applications.

## Describe the properties of a Branching Process.

## Demonstrate the knowledge and understanding of Queuing theory.

- Discuss the steady state probabilities
- Discuss the applications of the Birth and Death Queuing Models.

## Demonstrate understanding of stochastic models of the behavior of stock prices.

- Discuss the continuous time log-normal model of security prices and the empirical evidence for or against the model.

## Define and apply the main concepts of stochastic process Brownian motion (or Wiener Processes).

- Explain the definition and basic properties of standard Brownian motion or Wiener process.
- Write down the stochastic differential equation for geometric Brownian motion and show how to find its solution.
- Show an understanding of simple stochastic models for investment returns.
- Discuss the applications of pricing stock options.
- State the Arbitrage theorem.
- Discuss the applications of **geometric Brownian** motion.

## Describe the general techniques for simulating continuous random variables.

- Describe the generation of random variates from specified distribution.
- Discuss the inverse transformation method.
- Discuss the applications of acceptance and rejection methods.
- Disadvantages of using truly random numbers.
- Common sets of random numbers versus independent sets of random numbers.