

SYLLABUS

Semester I: 2018-2019 (181)

Instructor: Dr. A. Bonfoh
Course #: MATH 565
Title: Advanced Ordinary Differential Equations I

Textbook: Nonlinear Differential Equations and Dynamical Systems by F. Verhulst
(Second Edition, 1996. Revised 2006)

Objectives: The course aims to reinforce students' knowledge of the concepts of existence, uniqueness, continuation, asymptotic behavior and stability of solutions to ordinary differential equations.

Course description: Existence, uniqueness and continuity of solutions. Linear systems, solution space, linear systems with constant and periodic coefficients. Phase space, classification of critical points, Poincaré-Bendixson theory. Stability theory of linear and almost linear systems. Stability of periodic solutions. Lyapunov's direct method and applications.

Prerequisites: MATH 435 (Old 465)

Learning outcomes: Upon successful completion of this course, a student should be able to:

- Solve 1st order linear systems with constant coefficients.
- Prove existence, uniqueness and continuation of solutions to 1st order linear and nonlinear systems.
- Analyze the asymptotic behavior of solutions to linear, almost linear and periodic systems.
- Obtain phase-portrait of 2 and 3-dimensional autonomous systems.
- Analyze periodic solutions by applying the Poincaré-Bendixson theorem.
- Prove stability of solutions to linear, almost linear and periodic systems not only by the method of linearization but also by the Lyapunov's direct method.

Week	Date	Sec.	Topics	Suggested Homework Problems
1	Sep 2 – 6	1.2 1.3	Existence and uniqueness Gronwall's inequality	
2	Sep 9 – 13	2.1 2.2	Phase space, orbits Critical points and linearization	
3-4	Sep 16 – 27	2.3 2.4 2.5	Periodic solutions First integrals and integral manifolds Evolution of a volume element, Liouville's theorem	2.1, 2.2, 2.3, 2.5, 2.7, 2.8
5	Sep 30 – Oct 4	3.1 3.2 3.3	Two-dimensional linear systems Remarks on 3-dimensional linear systems Critical points of nonlinear equations	3.1, 3.3, 3.5, 3.6, 3.7
6	Oct 7 – Oct 11	4.1 4.2 4.3	Bendixson's criterion Geometric auxiliaries, preparation for the Poincaré-Bendixson theorem The Poincaré-Bendixson theorem	

First Exam: Wednesday, October 10, 2018 [1.2-4.1]				
7	Oct 14 – 18	4.4 4.5	Applications of the Poincaré-Bendixson theorem Periodic solutions in \mathbb{R}^n	4.2, 4.4, 4.5, 4.6, 4.7, 4.8
8	Oct 21 – 25	5.1 5.2	Simple examples Stability of equilibrium solutions	
9	Oct 28 – Nov 1 st	5.3 5.4	Stability of periodic solutions Linearization	5.1, 5.4, 5.5
10	Nov 4 – 8	6.1 6.2 6.3	Equations with constant coefficients Equations with coefficients which have a limit Equations with periodic coefficients	6.3, 6.5, 6.6, 6.7
11	Nov 11 – 15	7.1 7.2 7.3	Asymptotic stability of the trivial solution Instability of the trivial solution Stability of periodic solutions of autonomous equations	7.2, 7.3, 7.6, 7.7
Second Exam: Wednesday, November 14, 2018 [4.2-6.3]				
12	Nov 18 – 22	8.2 8.3	Lyapunov functions Hamiltonian systems and systems with first integrals	
13-14	Nov 25 – Dec 6	8.4	Applications and examples	8.1, 8.4, 8.7, 8.8, 8.9
15	Dec 9 – 13		Presentations	
Final Exam: Monday, December 17, 2018 [comprehensive]				

Grading:

Exam I, II	20% each
Homework assignments	20%
Presentations	10%
Final Exam	30%