KING FAHD UNIVERSITY OF PETROLEUM & MINERALS DEPARTMENT OF MATHEMATICS & STATISTICS

STAT 416: Stochastic Processes for Actuaries *(201)*

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Check Blackboard regularly for announcements

Textbook: Introduction to Probability Models, 11 h edition, by Sheldon M. Ross (2014)

Course Objectives:
Basic classes of stochastic processes. Poisson (regular, compound, compound surplus, and nonhomogenous) and renewal processes with applications in simple queuing systems and Actuarial Science. Discrete and continuous time Markov chains. Birth-Death and Yule processes. Branching models of population growth processes. Actuarial risk models; simulation. Arithmetic and geometric Brownian motions, and applications of these processes such as in computation of resident fees for continuing care retirement communities, and pricing of financial instruments.

Assessment for this course is based on **class activities (attendance & homework & Quizzes)**, ***a midterm exam*** and a ***comprehensive final exam***, as described in the following table:

Assessment

Activity	Weight
Attendance and participation	5%
6 Quizzes	30%
Home assignment	10%
First Major Exam: (chapters 4-5) **8 October 2020 (Class time)**	15%
Second Major Exam: (chapters 6-8) ***12 November 2020 (Class Time)***	*15%*
Final Exam: (Comprehensive) As posted on the Registrar Website	25%

Grade Assignment:

- You need to achieve at least 50% in order to pass the course

- Academic Integrity: All KFUPM policies regarding ethics and academic honesty apply to this course.

Important Notes:

- Excessive unexcused absences will result in a grade of DN in accordance with University rules.
- Attendance on time is very important

Home Work:

- Handout problems will be posted on blackboard.
- The Homework should be submitted in the first Saturday after completing the chapter and no need for an announcement in advance.
- No late homework will be accepted.

Missing an exam: No makeup exam will be given under any circumstances, when a student misses the midterm exam for a legitimate reason (such as medical emergency), his grade for this exam will be determined based on an existing formula, which depends on his performance in the non-missed exam and the final exam.

For ***Important Dates*** and ***Academic Calendar***, check the Registrar's site: **http://regweb.kfupm.edu.sa**

Syllabus – A rough weekly guideline

Week # (Dates)	Sections	Topics
Week 1 (Aug 30 – Sep 05)	1.1-1.6 2.1 – 2.9 3.1 – 3.5	Introduction to Probability theory (review) Random variables (review) Conditional Probability and Conditional Expectation (review)
Week 2 (Sep 06 – 12)	4.1 – 4.3	Introduction, Chapman – Kolmogorov Equation, Classification of States
Week 3 (Sep 13 – 19)	4.4-4.8	Limiting Probabilities, Some applications, Mean time Spent in Transient States, Branching Processes, Time Reversible Markov Chains,
Week 4 (Sep 20 – 26)	5.1 – 5.3	Introduction, The Exponential distribution and The Poisson Processes
Week 5 (Sep 27 – Oct 3)	5.3-5.4	The Poisson Processes (cont.) and Generalization of the Poisson Process

Week 6 (Oct 4 – 10)	6.1 – 6.4	Introduction Continuous-Time Markov Chains, Birth and Death Processes and The Transition Probability Function $Pij(t)$
Week 7 (Oct 11 – 17)	6.5-6.8	Limiting Probabilities, Time Reversibility, Uniformization and Computing the Transition Probabilities
Week 8 (Oct 18 – 24)	7.1-7.5	Introduction, Distribution of $N(t)$, Limit Theorems and their Applications, renewal Reward Processes and Regenerative Processes
Week 9 (Oct 25 – 31)	7.6-7.10	Semi – Markov Processes, the Inspection Paradox, Computing the Renewal Function, Applications to Patterns and The Insurance Ruin Problem
Week 10 (Nov 1 – 7)	8.1-8.7	Introduction, Preliminaries, Exponential Models, Network of Queues, The System M/G/1, Variation on the M/G/1 and the Model G/M/1
Week 11 (Nov 8 – 14)	9.1 – 9.3	Introduction, Structure Functions, Reliability of Systems of Independent Components and Bounds of the Reliability Function
Week 12 (Nov 15 – 21)	9.5-9.7	System Life as Function of Component Lives, Expected System Lifetime and System with Repair
Week 13 (Nov 22 – 28)	10.1- 10.4 &5-7	Brownian Motion, Hitting Times, Variations on Brownian Motion and Pricing stock Options
Week 14 (Nov 29 –Dec 05)	4.9, 11.1- 11.5	Markov-Chain Monte Carlo, Simulation (emphasis on 11.5 onwards)
Week 15 (Dec 06 – 12)	11.6- 11.8	Variance Reduction Techniques, Determining number of runs, Generating from Stationary distribution
Week 16 (Dec 13 – 14)		**Revision**

Learning objectives and outcomes:

On completion of the course the students will be able to:

Explain the concepts of probability:

- Explain what is meant by a set function, a sample space for an experiment, and an event.
- Define probability as a set function on a collection of events, stating basic axioms.
- Derive basic properties satisfied by the probability of occurrence of an event, and calculate probabilities of events in simple situations.
- Derive the addition rule for the probability of the union of two events, and use the rule to calculate probabilities.
- Define the conditional probability of one event given the occurrence of another event, and calculate such probabilities.
- Derive Bayes' Theorem for events, and use the result to calculate probabilities.

Understand the concepts of random variables.

- Define stochastic process.
- Use limit theorems.

Understand the concepts of conditioning and compounding.

- Define compound random variables.
- Compute expectations by conditioning.
- Use variance formula by conditioning.

Describe the properties of Poisson processes:

- For increments in the homogeneous case.
- For interval times in the homogeneous case.
- Resulting from special types of events in the Poisson process.
- Resulting from sums of independent Poisson processes.
- For any Poisson process and the interarrival and waiting distributions associated with the Poisson process.
- Calculate: a) Expected values b) Variances c) Probabilities. For a compound Poisson process, calculate moments associated with the value of the process at a given time.
- Discuss the properties of the nonhomogeneous Poisson process.
- Discuss the properties of the Compound Poisson process.
- Describe the properties of the exponential distribution.

Describe the properties of discrete and continuous Markov chains:

- For homogenous discrete-time Markov chain models: Define each model. Calculate probabilities of being in a particular state at a particular time.
- Calculate probabilities of transitioning between states.
- Explain what is meant by the Markov property in the context of a stochastic process.
- Define and apply a Markov chain.
- State the essential features of a Markov chain model.
- Applications of the Gambler's ruin problem.
- State the features of the Continuous-time Markov chains.

- Discuss the features of the Birth and death processes.
- Demonstrate how Markov chains can be used as a tool for modelling and how they can be simulated.
- Describe a system of frequency based experience rating in terms of a Markov chain and describe other simple applications.
- Describe a time-inhomogeneous Markov chain model and describe simple applications.

Describe the properties of a Branching Process.

Demonstrate the knowledge and understanding of Queuing theory:

- Discuss the steady state probabilities
- Discuss the applications of the Birth and Death Queuing Models.

Demonstrate understanding of stochastic models of the behavior of stock prices:

- Discuss the continuous time log-normal model of security prices and the empirical evidence for or against the model.

Define and apply the main concepts of stochastic process Brownian motion (or Wiener Processes):

- Explain the definition and basic properties of standard Brownian motion or Wiener process.
- Write down the stochastic differential equation for geometric Brownian motion and show how to find its solution.
- Show an understanding of simple stochastic models for investment returns.
- Discuss the applications of pricing stock options.
- State the Arbitrage theorem.
- Discuss the applications of geometric Brownian motion.

Describe the general techniques for simulating continuous random variables:

- Describe the generation of random variates from specified distribution.
- Discuss the inverse transformation method.
- Discuss the applications of acceptance and rejection methods.
- Disadvantages of using truly random numbers.
- Common sets of random numbers versus independent sets of random numbers.