King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
**SYLLABUS**
Semester II: 2020-2021 **(202)**

**Coordinator:** Dr. A. Bonfoh
**Course #:** MATH 568
**Title:** Advanced Partial Differential Equations I

**Textbook:** A basic course in Partial Differential Equations by Y. Qing Han, First Edition.

**References:** 1. Partial Differential Equations by L. C. Evans (Second Edition, 2010)
2. Beginning Partial Differential Equation. by P. O'Neil. (Second Edition, 2008)

**Objectives:** The course aims to reinforce students knowledge on concepts of existence, uniqueness and properties of solutions to first and second-order linear and quasilinear PDEs. Energy methods to solve nonlinear PDEs will be introduced. Applications to the wave equation, the heat equation and the Laplace equation are considered.

**Course description:** First order linear and nonlinear equations. Classification of Second order equations. The wave equation, heat equation and Laplace's equation. Green's functions, conformal mapping. Separation of variables, Sturm-Liouville theory. Maximum principles and regularity theorems.

**Prerequisites:** MATH 437

**Learning outcomes:** Upon successful completion of this course, a student should be able to:

- Solve quasilinear first order equation by the method of characteristics.
- Classify and solve $2^{nd}$ order PDE's by the method of characteristics.
- Solve the wave equation and analyze the well-posedness.
- Solve IBVP heat equation by using the maximum principle.
- Know the proofs of the representation theorems, MVP, and maximum principles for Laplace equation.
- Apply Green's function method and method of images to solve the Dirichlet and Neumann problems for the Laplace equation.
- Apply Energy methods to solve $2^{nd}$ order linear and nonlinear PDEs.

| Week | Date | Sec. | Topics | Suggested Homework Problems |
|---|---|---|---|---|
| 1 | Jan17 – 21 | | The linear first-order equation<br>The significance of characteristics | |
| 2 | Jan 24 – 28 | | The Quasilinear equations<br>Second order PDEs in two variables: classification | |
| 3 | Jan 31- Feb 4 | | The hyperbolic canonical form<br>The parabolic canonical form<br>The elliptic canonical form | |

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| 4 | Feb 7 – 11 | | The second-order Cauchy problem<br>Characteristics and the Cauchy problem<br>The wave equation : d'Alembert's solution of the the Cauchy problem<br>d'Alembert solution as a sum of waves | |
| 5 | Feb 14– 18 | | The characteristic triangle<br>The wave equation in 1-d<br>A nonhomogeneous problem in 1-d | |
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| 6 | Feb 21 – 25 | | A wave equation in 2-d<br>The Kirchoff-Poisson solution of the wave equation in 3-d<br>Hadamard's method of descent | |
| 7 | Feb 28-March 4 | | The heat equation: IBVP<br>The maximum principles | |
| 8 | March 7 – 11 | | The heat equation in 1-d<br>The nonhomogeneous heat equation in 1-d<br>The heat equation in 2-d | |
| **Midterm Exam** | | | | |
| 9 | March 14 – 18 | | Setting of Dirichlet and Neumann problems<br>Some harmonic functions<br>Representation theorems<br>Maximum principle, Mean value property | |
| 10 | March 21-25 | | Existence, Uniqueness and Well-posedness<br>Dirichlet problem in 2-d<br>Poisson's integral representation for a disk<br>Green's function for a Dirichlet problem | |
| 11-12 | March 28-April 8 | | The Neumann problem in 2-d<br>Sturm-Liouville theorems and regularity theorems | |
| 13-15 | April 11–29 | | Eigenvalue problem for the Laplace operator<br>Energy methods to solve 2nd order nonlinear PDEs | |
| **Final Exam [comprehensive]** | | | | |

**Grading:**

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| Midterm | 30% |
| Homework assignments | 30% |
| Presentation of Projects | 10% |
| Final Exam | 30% |